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A practical multiple model adaptive strategy for single-loop MPC

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Abstract

This paper details a multiple model adaptive control strategy for model predictive control (MPC). To maintain performance of this linear controller over a wide range of operating levels, a multiple model adaptive control strategy for dynamic matrix control (DMC), the process industry's standard for MPC, is presented. The method of approach is to design multiple linear DMC controllers. The tuning parameters for the linear controllers are obtained using novel analytical expressions. The controller output of the adaptive DMC controller is a weighted average of the multiple linear DMC controllers. The capabilities of the multiple model adaptive strategy for DMC are investigated through computer simulations and an experimental system.

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1. Introduction

Model predictive control (MPC) refers to a family of control algorithms that employ an explicit model to predict the future behavior of the process over an extended prediction horizon. These algorithms are formulated as a performance objective function, which is defined as a combination of set point tracking performance and control effort. This objective function is minimized by computing a profile of controller output moves over a control horizon. The first controller output move is implemented, and then the entire procedure is repeated at the next sampling instance. Fig. 1 illustrates the 'moving horizon' technique used in MPC.

Over the past decade, MPC has established itself in industry as an important form of advanced control (Richalet, 1993) due to its advantages over traditional controllers (García, Prett, & Morari, 1989; Muske & Rawlings, 1993). MPC displays improved performance because the process model allows current computations to consider future dynamic events. For example, this provides benefit when controlling processes with large dead times or nonminimum phase behavior. MPC

allows for the incorporation of hard and soft constraints directly in the objective function. In addition, the algorithm provides a convenient architecture for handling multivariable control due to the superposition of linear models within the controller.

Since the advent of MPC, various model predictive controllers have evolved to address an array of control issues (García et al., 1989; Froisy, 1994). Early forms used actual plant measurements and were based on an impulse or step response model (Richalet et al., 1978; Cutler & Ramaker, 1980). Additional modifications incorporated the need for on-line constraint handling (Morshedi, Cutler, & Skrovanek, 1985; García & Morshedi, 1986). A broad range of model-based MPC algorithms based on autoregressive moving average models emerged to address the issue of adaptation (e.g., Clarke, Mohtadi, & Tuffs, 1978a, b).

Dynamic matrix control (DMC) (Cutler & Ramaker, 1980) is the most popular MPC algorithm used in the chemical process industry today. Over the past decade, DMC has been implemented on a wide range of process applications (e.g., Li-wu & Corripio, 1985; McDonald & McAvoy, 1987; Goochee, Hatch, & Cadman, 1989; Hokanson, Houk, & Johnston, 1989; Tran & Cutler, 1989; Rovnak & Corlis, 1991; Maiti, Kapoor, & Saraf, 1994; Nikravesh, Farrell, Lee, & VanZee, 1995). A major part of DMC's appeal in industry stems from the use of a linear finite step response model of the process and a

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Nomenclature			
a_i	i th unit step response coefficient	y_{meas}	current measurement of the process variable
A_L	wall heat transfer area of the liquid	y_0	initial steady state of process variable
A_r	area	\hat{y}	predicted process variable
A	dynamic matrix	y_{sp}	process variable set point
c_i	i th term of the pseudo-inverse matrix	<i>Greek symbols</i>	
C_L	liquid heat capacity	Δu_i	change in controller output at the i th sample
C_v	valve coefficient	Δu_{1l}	change in controller output at the 1st sample at the l level of operation
d	disturbance prediction	Δu_{adap}	adapted controller output move
e	predicted error	$\Delta \bar{u}$	vector of controller output moves to be determined
\bar{e}	vector of predicted errors	γ_i^2	controlled variable weight (equal concern factor) in MIMO DMC
h	liquid height	$\Lambda^T \Lambda$	matrix of move suppression coefficients
h_L	liquid heat transfer coefficient	λ	move suppression coefficient (controller output weight)
i	index	λ_i^2	move suppression coefficients in MIMO DMC
I	identity matrix	$\Gamma^T \Gamma$	matrix of controlled variable weights
j	time index	Θ	time delay for the pH of the effluent stream
k	discrete dead time	θ_p	effective dead time of process
K_p	process gain	ρ_L	liquid density
l	level of operation index	τ_L	process lead time constant
M	control horizon (number of controller output moves)	τ_p	overall process time constant
n	current sample	τ_{p1}	1st process time constant
N	model horizon (process settling time in samples)	τ_{p2}	2nd process time constant
P	prediction horizon	<i>Abbreviations</i>	
pH ₄	effluent pH from the neutralization tank	DMC	dynamic matrix control
pK	log of the equilibrium constant	FOPDT	first order plus dead time
Q	flow rates	ITAE	integral of time weighted absolute error
R	number of measured outputs	IAE	integral of absolute error
S	number of manipulated inputs	MIMO	multiple-input multiple-output
S_L	cross-sectional area for liquid flow	MMAC	multiple model adaptive control
t	time	MPC	model predictive control
T	sample time	PID	proportional integral derivative
T_L	liquid temperature	POR	peak overshoot ratio
T_w	wall temperature	QDMC	quadratic dynamic matrix control
u	controller output variable		
W_{a4}, W_{b4}	reaction invariants of the effluent stream		
x	weighting factor		
y_l	value of the process variable at level l		

simple quadratic performance objective function. The objective function is minimized over a prediction horizon to compute the optimal controller output moves as a least-squares problem.

When DMC is employed on nonlinear chemical processes, the application of this linear model-based controller is limited to relatively small operating regions. Specifically, if the computations are based entirely on the model prediction (i.e. no constraints are active), the accuracy of the model has significant effect on the performance of the closed loop system (Gopinath, Bequette, Roy, Kaufman, & Yu, 1995). Hence, the

capabilities of DMC will degrade as the operating level moves away from its original design level of operation.

To maintain the performance of the controller over a wide range of operating levels, a multiple model adaptive control (MMAC) strategy for single loop DMC has been developed. The work focuses on a MMAC strategy for processes that are stationary in time, but nonlinear with respect to the operating level. In addition, this work does not address processes where the gain of the process changes sign.

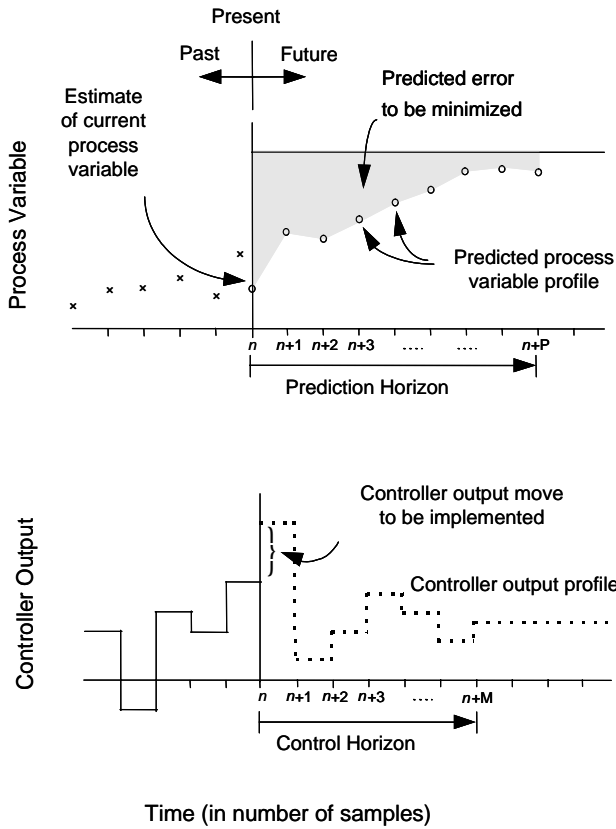


Fig. 1. The ‘moving horizon’ concept of model predictive control.

While this work is limited to single loop processes, one of the major benefits of DMC is in multivariable applications. The work presented here is important since it lays the foundation upon which a multivariable adaptive strategy can be constructed.

The method of approach is to construct a set of DMC process models that span the range of expected operation. By combining the process models to form a nonlinear approximation of the plant, the true plant behavior can be approached (Banerjee, Arkun, Ogunnaike, & Pearson, 1997).

The more models that are combined, the more accurate the nonlinear approximation will be. However, obtaining these models in industry can be expensive since the process must be perturbed from its desired level of operation. “Expensive” refers to the off-spec product produced when the system is perturbed along with the difficulties for the practitioner to obtain good data. Thus, the best number of DMC process models used in a particular implementation is a decision to be made by the designer on a case-by-case basis.

The novelty of this work lies in the details of the method. The approach involves combining multiple linear DMC controllers, each with their own step response model describing process dynamics at a specific level of operation. The final output forwarded to the

controller is obtained by interpolating between the individual controller outputs based on the value of the measured process variable. The tuning parameters for the linear controllers are obtained by using previously published tuning rules. The result is a simple and easy to use method for adapting the control performance without increasing the computational complexity of the control algorithm.

2. Background

2.1. Dynamic matrix control

DMC uses a linear finite step response model of the process to predict the process variable profile, $\hat{y}(n+j)$, over j sampling instants ahead of the current time, n :

$$\hat{y}(n+j) = y_0 + \underbrace{\sum_{i=1}^j a_i \Delta u(n+j-i)}_{\text{Effect of current and future moves}} + \underbrace{\sum_{i=j+1}^{N-1} a_i \Delta u(n+j-i)}_{\text{Effect of past moves}} \quad (1)$$

In Eq. (1), y_0 is the initial condition of the process variable, $\Delta u_i = u_i - u_{i-1}$ is the change in the controller output at the i th sampling instant, a_i is the i th unit step response coefficient of the process, and N is the model horizon and represents the number of sampling intervals of past controller output moves used by DMC to predict the future process variable profile.

The current and future controller output moves have not been determined and cannot be used in the computation of the predicted process variable profile. Therefore, Eq. (1) reduces to

$$\hat{y}(n+j) = y_0 + \sum_{i=j+1}^{N-1} (a_i \Delta u(n+j-i)) + d(n+j), \quad (2)$$

where the term $d(n+j)$ combines the unmeasured disturbances and the inaccuracies due to plant-model mismatch. Since future values of the disturbances are not available, $d(n+j)$ over future sampling instants is assumed to be equal to the current value of the disturbance, or

$$d(n+j) = d(n) = y(n) - y_0 - \sum_{i=1}^{N-1} (a_i \Delta u(n-i)), \quad (3)$$

where $y(n)$ is the current process variable measurement.

The goal is to compute a series of controller output moves such that

$$y_{sp}(n+j) - \hat{y}(n+j) = 0 \quad j = 1, 2, \dots, P, \quad (4)$$

where P is the prediction horizon and represents the number of sampling intervals into the future over which

DMC predicts the future process variable. Substituting Eq. (1) in Eq. (4) gives

$$\begin{aligned}
 & \underbrace{y_{sp}(n+j) - y_0 - \sum_{i=j+1}^{N-1} a_i \Delta u(n+j-i) - d(n)}_{\text{Predicted error based on past moves, } e(n+j)} \\
 &= \underbrace{\sum_{i=1}^j a_i \Delta u(n+j-i)}_{\text{Effect of current and future moves to be determined}} \\
 & j = 1, 2, \dots, P. \tag{5}
 \end{aligned}$$

Eq. (5) is a system of linear equations that can be represented as a matrix equation of the form

$$\begin{aligned}
 & \begin{bmatrix} e(n+1) \\ e(n+2) \\ e(n+3) \\ \vdots \\ e(n+M) \\ \vdots \\ e(n+P) \end{bmatrix}_{P \times 1} \\
 &= \begin{bmatrix} a_1 & 0 & 0 & \dots & 0 \\ a_2 & a_1 & 0 & & 0 \\ a_3 & a_2 & a_1 & \ddots & 0 \\ \vdots & \vdots & \vdots & & 0 \\ a_M & a_{M-1} & a_{M-2} & & a_1 \\ \vdots & \vdots & \vdots & & \vdots \\ a_P & a_{P-1} & a_{P-2} & \dots & a_{P-M+1} \end{bmatrix}_{P \times M} \\
 & \times \begin{bmatrix} \Delta u(n) \\ \Delta u(n+1) \\ \Delta u(n+2) \\ \vdots \\ \Delta u(n+M-1) \end{bmatrix}_{M \times 1} \tag{6}
 \end{aligned}$$

or in a compact matrix notation as

$$\bar{e} = A \Delta \bar{u}, \tag{7}$$

where \bar{e} is the vector of predicted errors over the next P sampling instants, A is the dynamic matrix, and $\Delta \bar{u}$ is the vector of controller output moves to be determined.

An exact solution to Eq. (7) is not possible since the number of equations exceeds the degrees of freedom ($P > M$). Hence, the control objective is posed as a least-squares optimization problem with a quadratic performance objective function of the form

$$\underset{\Delta \bar{u}}{\text{Min}} J = [\bar{e} - A \Delta \bar{u}]^T [\bar{e} - A \Delta \bar{u}]. \tag{8}$$

In the unconstrained case, this minimization problem has a closed form solution, which represents

the DMC control law:

$$\Delta \bar{u} = (A^T A)^{-1} A^T \bar{e}. \tag{9}$$

Implementation of DMC with the control law in Eq. (9) results in excessive control action, especially when the control horizon is greater than one. Therefore, a quadratic penalty on the size of controller output moves is introduced into the DMC performance objective function. The modified objective function has the form

$$\underset{\Delta \bar{u}}{\text{Min}} J = [\bar{e} - A \Delta \bar{u}]^T [\bar{e} - A \Delta \bar{u}] + [\Delta \bar{u}]^T \lambda [\Delta \bar{u}], \tag{10}$$

where λ is the move suppression coefficient. In the unconstrained case, the modified objective function has a closed form solution of (e.g., Marchetti, Mellichamp, & Seborg, 1983; Ogunnaike, 1986)

$$\Delta \bar{u} = (A^T A + \lambda I)^{-1} A^T \bar{e}. \tag{11}$$

Adding constraints to the classical formulation given in Eq. (10) produces the quadratic dynamic matrix control (QDMC) (Morshedi et al., 1985; García & Morshedi, 1986) algorithm. The constraints considered in this work include:

$$\hat{y}_{\min} \leq \hat{y} \leq \hat{y}_{\max}, \tag{12a}$$

$$\Delta \bar{u}_{\min} \leq \Delta \bar{u} \leq \Delta \bar{u}_{\max}, \tag{12b}$$

$$\bar{u}_{\min} \leq \bar{u} \leq \bar{u}_{\max}. \tag{12c}$$

2.2. Adaptive mechanisms

Several excellent technical reviews of adaptive control mechanisms recount the various approaches for controlling nonlinear processes from both an academic and an industrial perspective (Seborg, Edgar, & Shah, 1986; Bequette, 1991; Di Marco, Semino, & Brambilla, 1997). In addition, Qin and Badgwell (2000) provide a good overview of nonlinear MPC applications that are currently used in industry. As illustrated by these works, adding an adaptive mechanism to MPC has been approached a number of ways. Researchers have primarily focused on updating the internal process model. These include the use of a nonlinear analytical model, combinations of linear empirical models or some combination of both. There have been less developments focusing on updating the tuning parameters.

2.3. Nonlinear analytical modeling

In general, analytical models are difficult to obtain due to the underlying physics and chemistry of the process. In addition, they are often too complex to employ directly in the optimization calculation.

García (1984), for example, extends the basic QDMC formula to handle nonlinear processes by employing the nonlinear analytical equations directly in the control algorithm. The projected process variable profile is calculated by integrating the nonlinear, ordinary differential equations of the model over the prediction horizon, while keeping the controller output constant. The main assumption in this version of nonlinear DMC is that the model coefficients remain constant while each control move is calculated. Therefore, the dynamic matrix can be used for predicting the controller output profile. At each sampling instance, a linear model is obtained by linearization of the nonlinear model, and this linear model is used to calculate the step response coefficients used in the next prediction step.

In a similar method, Krishnan and Kosanovich (1998) developed a multiple model predictive controller by linearizing the nonlinear model of the process around the process variables' reference trajectories. Another popular approach is to linearize the nonlinear analytical equations around the current measurement of the process variable at each sampling instance to obtain linear discrete state space equations (Gattu & Zafriou, 1992, 1995; Lee & Ricker, 1994; Gopinath et al., 1995). The states of the process are then estimated based on recursive identification techniques that involve the use of a Kalman filter.

The method by Lakshmanan and Arkun (1999) uses the nonlinear analytical model to obtain linear state space models at different operating levels. The internal DMC process model is updated by weighting the linear models by using a Bayesian estimator that is based on a past window of measurement data. A simplification of this method is to employ a nonlinear convolution model. The internal DMC process model is divided into a linear dynamic part which consists of the process time constants and process dead times and a nonlinear steady state part which consists of the process gains. The nonlinear steady state part is then developed from the nonlinear analytical process model (Bodizs, Szeifert, & Chovan, 1999).

In addition, simple nonlinear output transformations have been applied to the nonlinear analytical equations in order to linearize the process model (Georgiou, Georgakis & Luyben, 1988). This method improves the performance of DMC for nonlinear processes. However, it is highly system dependent since the transformations are developed based on the analytical models. In addition, output transformations can be difficult to design for some chemical processes.

Some adaptive strategies use the nonlinear analytical model directly in the algorithm. In these methods, the performance objective functions are modified in order to incorporate the nonlinear model either directly in the objective function or as process constraints (Ganguly & Saraf, 1993; Sistu, Gopinath, & Bequette, 1993;

Katende, Jutan, & Corless, 1998; Xie, Zhou, Jin, & Xu, 2000).

Peterson, Hernández, Arkun, and Schork (1992) calculate an estimate of the disturbance as a combination of the external disturbances and the nonlinearities in the process. Hence, the disturbance becomes nonlinear and time varying, enabling the DMC step response model to remain in traditional form.

Other researchers (e.g., Gundala, Hoo, & Piovoso, 2000) used a combination of both multiple non-adaptive and adaptive models to control the nonlinear process by switching or weighting the models. The control structure is based on a model reference adaptive controller.

2.4. *Combinations of linear empirical models*

Recursive formulations are used on-line to update the parameters of the process model as new plant measurements become available at each sampling instance (McIntosh, Shah, & Fisher, 1991; Maiti et al., 1994; Maiti, Kapoor, & Saraf, 1995; Ozkan & Camurdan, 1998; Liu & Daley, 1999; Yoon, Yang, Lee, & Kwon, 1999; Zou & Gupta, 1999; Chikkula & Lee, 2000). A number of problems can arise from employing recursive estimation schemes. These include: convergence problems if the data does not contain sufficient and persistent excitation and inaccurate model parameters if unmeasured disturbances or noise influence the measurements. In addition, recursive methods may be sensitive to process dead times and high noise levels.

A more practical adaptive strategy uses a gain and time constant schedule for updating the process model (McDonald & McAvoy, 1987; Chow, Kuznetsoc, & Clarke, 1998). An extension of this method is to use multiple models to update the process model. Linear models that described the system at various operating points are developed based on plant measurements. Past researchers (e.g., Banerjee et al., 1997) have illustrated that linear models can be combined in order to obtain an approximation of the process that approaches its true behavior. Two different multiple model methods can be employed to maintain the performance of the controller over all operating levels.

In one case, a controller is designed for each level of operation. This approach has been applied to generalized predictive control and proportional-integral-derivative controllers. The controller moves are weighted based on the prediction error calculated for the individual controllers. The resulting weights are obtained using recursive identification such that the prediction error is minimized (Yu, Roy, Kaufman, & Bequette, 1992; Schott & Bequette, 1994; Townsend & Irwin, 2001).

Although the concept used in this paper is similar to those listed above, there are important differences. One of the differences of this approach is that the strategy is

applied directly to the DMC algorithm. The method of approach is to design and combine multiple linear DMC controllers, each with their own step response model. Another contribution is that the proposed methodology does not introduce additional computation complexity.

For the other case, a single controller is used. Even though this concept is not used in the proposed method, the strategy is related. Gendron et al. (1993) developed a multiple model pole placement controller. The process models are weighted based on the current process variable measurement. The weighted model is then used in a single pole placement controller. Rao, Aufderheide, and Bequette (1999) and Townsend and Irwin (2001) designed a multiple model adaptive model predictive controller. The set of process models are weighted based on the prediction error. The weighted model is then set to a single controller.

Townsend, Lightbody, Brown, and Irwin (1998) developed a nonlinear DMC controller that replaces the linear process model with a local model network. This local model network contains local linear ARX models and is trained using a hybrid learning technique. From this local model network, the DMC controller is supplied with a weighted step response model.

Chang, Wang, and Yu (1992) averages two linear step response models that are obtained at different operating levels to arrive at a single step response model. This average process model is then used directly in the DMC algorithm.

3. Formulation of a MMAC strategy for DMC

The method of approach in this work focuses on updating the DMC controller output move based on a minimum of three local linear models that span the range of operation. Three linear models are used to make this adaptive strategy more functional to the practitioner since collecting plant data is difficult and time consuming. In addition, by using three linear models it is possible to achieve a timely response since the computational burden associated with convergence and parameters updates is avoided. The scope of this work is limited to processes that are stationary in time but nonlinear with respect to the operating level.

3.1. Non-adaptive DMC implementation

The foundation of this strategy lies with the formal tuning rules for non-adaptive DMC (Shridhar & Cooper, 1997, 1998) based on fitting the controller output to measured process variable dynamics at one level of operation with a FOPDT model approximation.

A FOPDT model has the form

$$\tau_p \frac{dy(t)}{dt} + y(t) = K_p u(t - \theta_p) \quad \text{or} \quad \frac{y(s)}{u(s)} = \frac{K_p e^{-\theta_p s}}{\tau_p s + 1}, \quad (13)$$

where K_p is the process gain, τ_p is the overall time constant and θ_p is the effective dead time.

Although a FOPDT model approximation does not capture all the features of higher order processes, it often reasonably describes the process gain, overall time constant and effective dead time of such processes (Cohen & Coon, 1953). Specifically, K_p indicates the size and direction of the process variable response to a control move, τ_p describes the speed of the response, and θ_p tells the delay prior to when the response begins. In the past, tuning strategies based on a FOPDT model such as Cohen-Coon, IAE and ITAE have proved useful for PID implementations. Previous research for tuning DMC (Shridhar & Cooper, 1997, 1998) has demonstrated that this limited amount of information is sufficient to achieve desirable closed loop DMC performance at the specified design level of operation.

The tuning parameters for single-loop DMC include:

- the sample time, T
- finite prediction horizon, P
- model horizon (process settling time in samples), N
- control horizon (number of controller output moves that are computed), M
- move suppression coefficient (controller output weight), λ

The tuning parameters and the step response coefficients are calculated offline prior to the start-up of the non-adaptive DMC controller. Following this previous work, the sample time, T , is computed as

$$T = \text{Max}(0.1\tau_p, 0.5\theta_p). \quad (14)$$

This value of sample time balances the desire for a low computation load (a large T) with the need to properly track the evolving dynamic behavior (a small T). Many control computers restrict the choice of T (e.g., Franklin & Powell, 1980; Åström & Wittenmark, 1984). Recognizing this, the remaining tuning rules permit values of T other than that computed by Eq. (14) to be used.

The sample time and the effective dead time are used to compute the discrete dead time in integer samples as

$$k = \text{Int}\left(\frac{\theta_p}{T}\right) + 1. \quad (15)$$

The prediction horizon, P , and the model horizon, N , are computed as the process settling time in samples as

$$P = N = \text{Int}\left(\frac{5\tau_p}{T}\right) + k. \quad (16)$$

Note that both N and P cannot be selected independent of the sample time.

A larger P improves the nominal stability of the closed loop. For this reason, P is selected such that it includes the steady-state effect of all past controller output moves, i.e., it is calculated as the open loop settling time of the FOPDT model approximation.

In addition, it is important that N be equal to the open loop settling time of the process to avoid truncation error in the predicted process variable profile. Eq. (16) computes N as the settling time of the FOPDT model approximation. This value is long enough to avoid the instabilities that can otherwise result since truncation of the model horizon misrepresents the effect of past controller output moves in the predicted process variable profile (Lundström, Lee, Morari, & Skogestad, 1995).

The control horizon, M , must be long enough such that the results of the control actions are clearly evident in the response of the measured process variable. The tuning rule thus chooses M as one dead time plus one time constant, or

$$M = \text{Int}\left(\frac{\tau_p}{T}\right) + k. \quad (17)$$

This equation calculates M such that $M \times T$ is larger than the time required for the FOPDT model approximation to reach 60% of the steady state.

The final step is the calculation of the move suppression coefficient, λ . Its primary role in DMC is to suppress aggressive controller actions. Shridhar and Cooper (1997, 1998) derived the move suppression coefficient based on a FOPDT model fit as

$$\lambda = \frac{M}{10} \left(\frac{3.5\tau_p}{T} + 2 - \frac{(M-1)}{2} \right) K_p^2. \quad (18)$$

Eq. (18) is valid for a control horizon greater than 1 ($M > 1$). When the control horizon is 1 ($M = 1$), no move suppression coefficient should be used ($\lambda = 0$).

With the tuning parameters determined, the step response coefficients, a_1, a_2, \dots, a_N , are calculated. The dynamic matrix, A , is then formulated using the first P step response coefficients:

$$A = \begin{bmatrix} a_1 & 0 & 0 & \cdots & 0 \\ a_2 & a_1 & 0 & & 0 \\ a_3 & a_2 & a_1 & \ddots & 0 \\ \vdots & \vdots & \vdots & & 0 \\ a_M & a_{M-1} & a_{M-2} & & a_1 \\ \vdots & \vdots & \vdots & & \vdots \\ a_P & a_{P-1} & a_{P-2} & \cdots & a_{P-M+1} \end{bmatrix}_{P \times M} \quad (19)$$

permitting the evaluation of the control matrix:

$$(A^T A + \lambda I)^{-1} A^T, \quad (20)$$

where I is an $M \times M$ identity matrix.

Now, at each sample time, the current and future predicted process variable profile is computed,

$$\hat{y}(n+j) = y_o + \sum_{i=j+1}^N a_i \underbrace{(u(n+j-i) - u(n+j-i-1))}_{\text{Change in controller output } \Delta u(n+j-i)}, \quad (21)$$

and the values of the disturbance vector are estimated as

$$d(n+j) = d(n) = y_{sp}(n) - \hat{y}(n). \quad (22)$$

From Eqs. (21) and (22), the predicted error is computed as

$$\bar{e} = \begin{bmatrix} y_{sp}(n+1) - \{\hat{y}(n+1) + d(n+1)\} \\ y_{sp}(n+2) - \{\hat{y}(n+2) + d(n+2)\} \\ y_{sp}(n+3) - \{\hat{y}(n+3) + d(n+3)\} \\ \vdots \\ y_{sp}(n+P) - \{\hat{y}(n+P) + d(n+P)\} \end{bmatrix}_{P \times 1}. \quad (23)$$

Let c_i denote the i th first row element of the pseudo-inverse matrix, $(A^T A + \lambda I)^{-1} A^T$. Using Eq. (23), the current controller output move that results is

$$\Delta u_1 = \underbrace{[c_1 c_2 \cdots c_P]_{1 \times P}}_{\text{DMC gain vector}} \bar{e}_{P \times 1}. \quad (24)$$

3.2. The adaptive strategy

The adaptive DMC strategy exploits the non-adaptive formal tuning rule and the DMC control move calculation. For clarity, the approach for the adaptive strategy presented here involves designing and combining three non-adaptive DMC controllers. However, the method can involve designing and combining any number of non-adaptive controllers.

As explained below, all use the same values for T, P, N , and M , while λ varies for each controller. The three controllers each compute their own control action. These are then weighted and combined to yield a single control move forwarded to the final control element.

Although three controllers are employed in this work, the approach can easily be expanded to include as many local linear controllers as the practitioner would like. The use of three linear DMC controllers is the minimum needed to adequately control a nonlinear process. The more linear controllers that are used, the better the adaptive controller will perform. While this method will often not capture the severe nonlinear behaviors associated with many processes, it will provide significant improvement over non-adaptive DMC.

Implementation begins by collecting three sets of step test data, at a lower, middle and upper level of the expected operating range. Each of the models should describe the process around the point in which the data

was collected. Two of the step test data sets should be collected at the upper and lower extremes of the expected operating region to ensure that the nonlinear approximation reasonably describes the actual process over the entire operating range (Di Marco et al., 1997). The third set of step test data should be obtained around the middle of the expected operating region. Operating level is defined as a specific value for the measured process variable, y_l , where $l = 1, 2, 3$ are for the lower, middle and upper level of operation, respectively.

Each data set is fit with a linear FOPDT model for use in the tuning correlations. Step response coefficients for the internal DMC process model as shown in Eq. (6) are generated by introducing a series of positive and negative steps in the controller output with the process at steady state and the controller in manual mode. From the instant the first step change is made, the process variable response is recorded as it evolves and settles at a new steady state. Note that the closed-loop data can also be used to generate the step response coefficients by stepping the set point of the controller and recording the response of the measured process variable and controller output. For a step in the controller output of arbitrary size, the response data is normalized by dividing through by the size of the controller output step to yield the unit step response. This is performed for each operating level, and it is necessary to make the controller output step large enough such that noise in the process variable measurement does not mask the true process behavior.

The tuning parameters for the adaptive DMC strategy are computed by employing the formal tuning rules given in Eqs. (14)–(18). Tuning parameters are calculated for each of the l data sets.

Recall that all three controllers use the same value of T , P , N , and M . Here, T is selected as close as possible to the smallest T_l from the three data sets, or

$$T = \text{Min}(T_l). \quad (25)$$

This ensures that when the process is operating in the level with the fastest dynamics, the sample time is fast enough to capture the process behavior. Since many control computers restrict the choice of T (e.g., Franklin & Powell, 1980; Åström & Wittenmark, 1984), the remaining tuning rules permit values of T other than that computed by Eq. (25) to be used.

Once the sample time is selected, the tuning parameters P , N , and M needed to be recalculated for each of the l data sets as

$$P_l = N_l = \text{Int}\left(\frac{5\tau_{pl}}{T}\right) + k_l \text{ where } k_l = \text{Int}\left(\frac{\theta_{pl}}{T}\right) + 1, \quad (26a)$$

$$M_l = \text{Int}\left(\frac{\tau_{pl}}{T}\right) + k_l. \quad (26b)$$

The adaptive tuning parameters P , N , and M are selected as the maximum values:

$$P = \text{Max}(P_l), \quad (27a)$$

$$N = \text{Max}(N_l), \quad (27b)$$

$$M = \text{Max}(M_l). \quad (27c)$$

Thus, the horizons will always be long enough to capture the slowest dynamic behaviors in the range of operation.

Even though the above tuning parameters remain fixed upon implementation, success in this adaptive strategy requires that λ vary based upon each data set. Since each data set will have different values for K_p , τ_p and θ_p , the value of λ_l calculated for each data set must reflect this difference, or

$$\lambda_l = \frac{M}{10} \left(\frac{3.5\tau_{pl}}{T} + 2 - \frac{(M-1)}{2} \right) K_{pl}^2. \quad (28)$$

Note that the calculation of λ is based upon M and not M_l . This allows λ to suppress aggressive control actions over the entire control horizon. Similar to non-adaptive DMC, Eq. (28) is valid for a control horizon greater than 1 ($M > 1$), and if the control horizon is 1 ($M = 1$), then no move suppression coefficient is used ($\lambda_l = 0$).

Upon implementation, the MMAC strategy for DMC calculates three non-adaptive DMC controller output moves, one for each level of operation as defined by the test data sets. The adaptive controller output move, Δu_{adap} , is a weighted average of each controller output move

$$\Delta u_{adap} = \sum_{l=1}^3 x_l \Delta u_{l1}, \quad (29)$$

where x_l is a weighting factor. If y_{meas} is the actual value of the measured process variable at the current sample time, then

If $y_{meas} \geq y_3$ then

$$x_1 = 0; \quad x_2 = 0; \quad x_3 = 1. \quad (30)$$

If $y_2 < y_{meas} < y_3$ then

$$x_1 = 0; \quad x_2 = 1 - x_3; \quad x_3 = \frac{y_{meas} - y_2}{y_3 - y_2}. \quad (31)$$

If $y_1 < y_{meas} < y_2$ then

$$x_1 = 1 - x_2; \quad x_2 = \frac{y_{meas} - y_1}{y_2 - y_1}; \quad x_3 = 0. \quad (32)$$

If $y_{meas} \leq y_1$ then

$$x_1 = 1; \quad x_2 = 0; \quad x_3 = 0. \quad (33)$$

In the event that $y_{meas} = y_2$, then the adaptive controller output move equals the value associated with the middle data set. Hence, the weight factors are in the range of [0,1]. The value of the adaptive controller output finally implemented is calculated as

$$u(n) = u(n-1) + \Delta u_{adapt}. \quad (34)$$

4. Demonstration of single-loop adaptive DMC

The adaptive DMC algorithm is demonstrated on three process simulations, a transfer function model, a heat exchanger and a pH neutralization process. The fourth demonstration of the adaptive DMC algorithm is for the gravity drained tanks experiment at the University of Connecticut.

4.1. Transfer function model

Three different transfer functions are combined to form a nonlinear model. The general form of each transfer function is

$$G_p(s) = \frac{K_p(\tau_{L2}s + 1)e^{-\theta_P s}}{(\tau_{P1}s + 1)(\tau_{P2}s + 1)}. \quad (35)$$

Each of the three transfer functions has different parameter values, and each exactly describes the behavior of the process at a specific value of the measured process variable. At intermediate values of the measured process variable, the transfer function contributions are combined using a linear weighting function to yield a continually changing dynamic behavior.

Table 1 lists the parameters used for each of the three transfer functions. As listed in the table, a model is defined at a measured process variable value of 20%, 50%, and 80%. Note that each parameter in the table changes by a factor of 3 from the lower to upper level of operation, except the process gain which changes by a factor of 6.

Dynamic tests are performed by pulsing the controller output at each level of operation, yielding three sets of test data. Following the adaptive DMC design procedure described previously, a FOPDT model is fit to each data set to yield the parameters listed in Table 1. The FOPDT parameters are then used in Eqs. (14)–(18) to obtain the non-adaptive DMC tuning parameters also listed.

Table 2 lists the tuning parameters for the adaptive DMC strategy obtained by using Eqs. (25)–(28). Note that as described in the adaptive strategy, all three controllers use the same value of T , P , N , and M , while λ varies. This ensures that the sample time is short enough to capture the fastest dynamic behaviors while the

Table 1

General parameters, FOPDT parameters and DMC tuning parameters for the transfer function model

	Lower level	Middle level	Upper level
Process variable value (%)	20	50	80
SOPDT with lead time model parameters			
K_P	1	3	6
τ_{P1} (time units)	10	20	30
τ_{P2} (time units)	5	10	15
τ_L (time units)	–15	–10	–5
θ_P (time units)	3	6	9
FOPDT model fit parameters			
K_P	1.20	3.13	6.12
τ_P (time units)	15.3	25.3	37.7
θ_P (time units)	15.9	21.2	23.2
DMC tuning parameters			
T (time units)		11	
P (samples)		13	
N (samples)		13	
M (samples)		4	
λ		31.6	

Table 2

Adaptive DMC tuning parameters for the transfer function model

	Lower level	Middle level	Upper level
Process variable value (%)	20	50	80
T (time units)	8	8	8
P (samples)	26	26	26
N (samples)	26	26	26
M (samples)	7	7	7
λ	5.2	63.6	391.4

horizons are long enough to capture the slowest dynamic behaviors in the range of operation.

The control objective in this study is set point tracking across the range of nonlinear operation. The design goal is a fast rise time with a 2% peak overshoot ratio (POR).

Non-adaptive DMC uses the tuning parameters associated with the middle level of operation (i.e. the measured process variable equals 50%). It is reasonable to design the non-adaptive controller based on the middle level of operation because this will yield a compromise in performance over the range of dynamic behaviors.

Fig. 2 shows the response of the process variable for both the non-adaptive and adaptive DMC implementations. As illustrated by the figure, the performance of the non-adaptive DMC varies greatly as the dynamic behavior of the process changes. As the set point is stepped across the range of operation, the performance of the non-adaptive controller varies from an under-damped response to one that is over-damped and

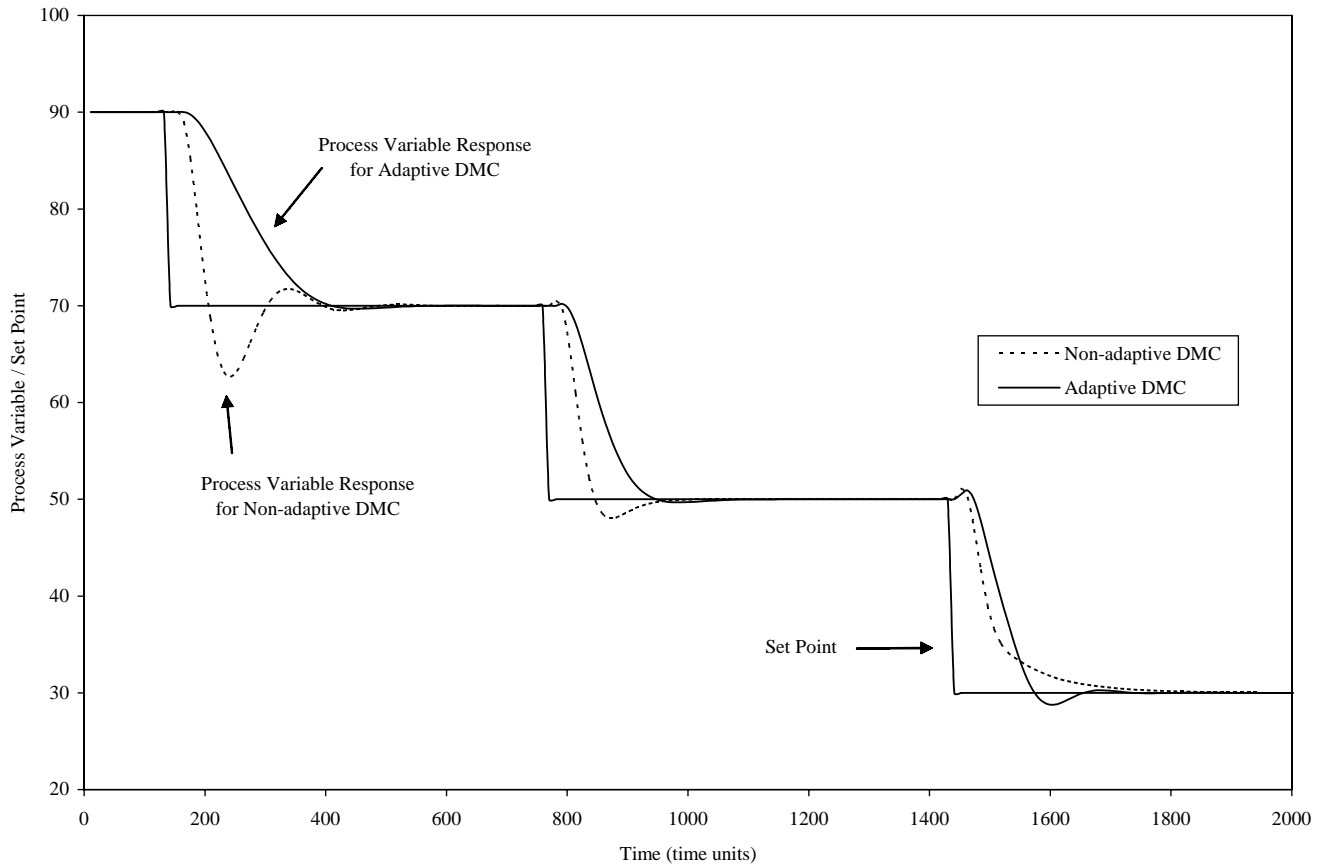


Fig. 2. Response of the process variable for the transfer function model using non-adaptive and adaptive DMC.

sluggish in nature. The adaptive strategy, on the other hand, is able to maintain the design performance over the entire operating region.

In particular, the response of the process variable for the non-adaptive controller exhibits a POR of 35% for the set point step from 90% to 70% and a POR of 10% for the set point step from 70% to 50%. For the set point step from 50% to 30%, the non-adaptive controller displays a sluggish response with no overshoot. The adaptive controller was able to substantially maintain the 2% POR with consistent rise time across the entire range.

4.2. Heat exchanger

The heat exchanger, shown in Fig. 3, is a counter-current, shell and tube, lube oil cooler. This simulation is one of the case studies available in Control Station[®]. Control Station is a controller design and tuning tool and a process control training simulator used by industry and academic institutions worldwide for control loop analysis and tuning, dynamic process modeling and simulation, performance and capability studies, hands-on process control training. More information

and a free demo are available at www.controlstation.com.

The general heat exchanger model is described using a shell energy balance as

$$\rho_L C_L S_L \frac{\partial T_L}{\partial t} = -\rho_L C_L S_L v \frac{\partial T_L}{\partial z} + h_L A_L (T_w - T_L). \quad (36)$$

In the simulation studied here, physical properties are assumed constant. The partial differential equation, Eq. (36), is implemented using a lumped parameter approach. Specifically, the simulation is modeled as five counter-current continuously stirred tank reactors with heating coils. For more details, see Stauffer (2001).

The controller output manipulates the flow rate of cooling water on the shell side. The measured process variable is the lube oil temperature exiting the exchanger on the tube side. This process displays a nonlinear behavior in that the process gain changes by a factor of 5 over the range studied in this example.

Three sets of test data were obtained at exit temperatures (measured process variables) of 130°C, 145°C, and 160°C. Dynamic tests are performed by pulsing the controller output at each level of operation, generating three sets of test data. Following the procedure just described in the previous example, each

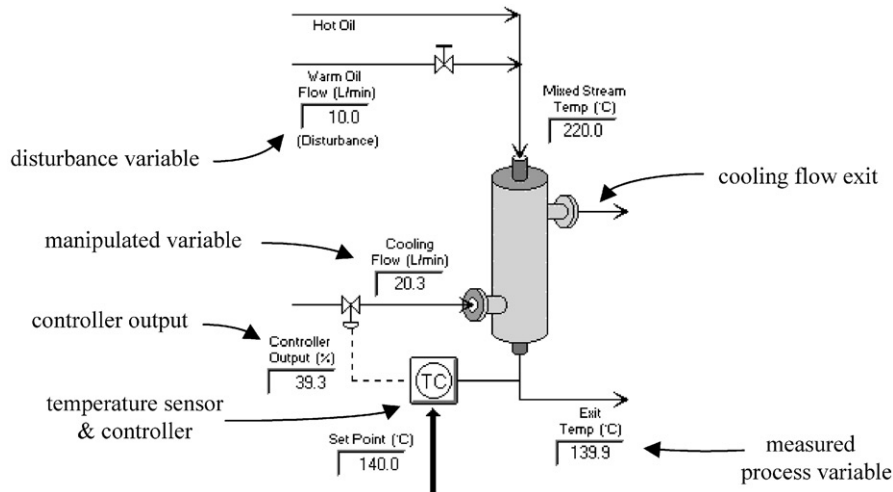


Fig. 3. Heat Exchanger Graphic from Control Station[®] Software Package.

data set is fit with a FOPDT model (results listed in Table 3) and these parameters are used to compute the adaptive DMC tuning values (results listed in Table 4).

Process constraints were included so as to compare non-adaptive and adaptive QDMC. The constraints considered in this investigation include:

$$130 \leq \hat{y} \leq 170, \tag{37a}$$

$$-5 \leq \Delta \bar{u} \leq 5, \tag{37b}$$

$$0 \leq \bar{u} \leq 100. \tag{37c}$$

While other choices for the constraints are possible, it was found that the benefit of the adaptive strategy remained apparent for a wide range of constraint values.

The control objective was set point tracking capabilities across the entire range of operation. The design goal for this study is a fast rise time with a 10% POR. Non-adaptive QDMC employs the tuning parameters associated with the middle level of operation (i.e. the measured process variable equals 145°C).

Fig. 4 displays the response of the process variable for both the non-adaptive and adaptive QDMC implementations. As the set point is stepped from 130°C to 170°C the behavior of the process variable for non-adaptive DMC ranges from a response that is over-damped to a response that is under-damped. As the process reaches higher temperatures, the process variables response for the non-adaptive QDMC controller becomes more oscillatory with longer settling times.

Specifically, as the set point is stepped from 130°C to 140°C, the response of the process variable for the non-adaptive controller displays a sluggish rise time with no POR. For the set point step from 140°C to 150°C, the controller is able to maintain the design goal since the non-adaptive controller was designed around this level

Table 3
FOPDT parameters and DMC tuning parameters for the heat exchanger

	Lower level	Middle level	Upper level
Process variable value (°C)	130	145	160
FOPDT model parameters			
K_P (°C/%)	-0.3	-0.8	-1.6
τ_P (min)	0.9	1.1	1.2
θ_P (min)	0.8	0.8	0.9
DMC tuning parameters			
T (s)		24	
P (samples)		16	
N (samples)		16	
M (samples)		5	
λ		3.1	

Table 4
Adaptive DMC tuning parameters for the heat exchanger

	Lower level	Middle level	Upper level
Process variable value (°C)	130	145	160
T (s)	24	24	24
P (samples)	17	17	17
N (samples)	17	17	17
M (samples)	5	5	5
λ	0.3	3.1	12.6

of operation. The response of the process variable for the non-adaptive controller exhibits a POR of 40% for the set point step from 150°C to 160°C and a POR of 75% for the set point step from 160°C to 170°C.

The adaptive QDMC controller displayed no problems in maintaining the design goal of a fast rise time with a 10% POR over the expected range of operation.

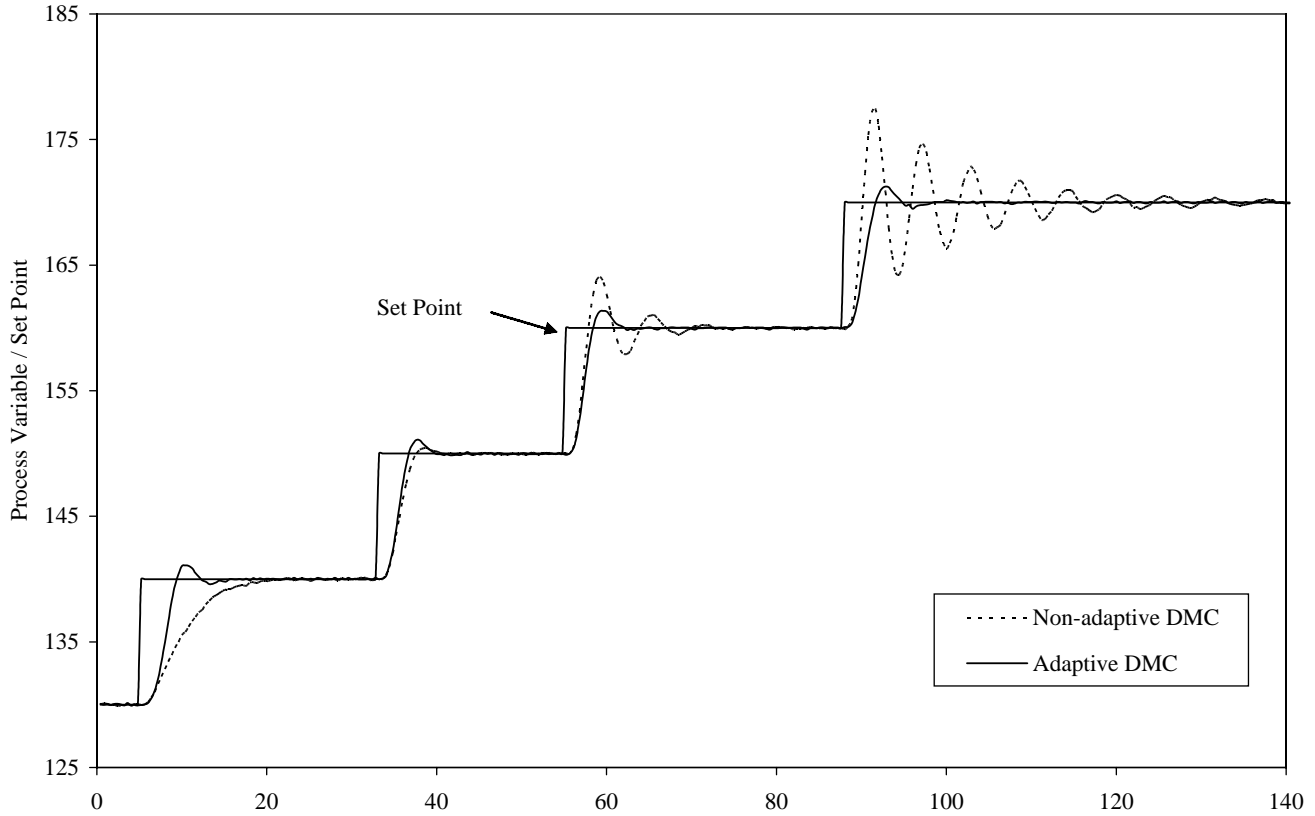


Fig. 4. Response of the process variable for the heat exchanger simulation using non-adaptive and adaptive QDMC.

As evidenced by the graph, even if the set point is stepped outside of the expected operating range, the performance of the adaptive strategy does not degrade.

4.3. pH neutralization process

A schematic diagram of the pH neutralization process is shown in Fig. 5. The neutralization process represents a highly nonlinear process. The dynamic model used in this work is representative of the experimental pH neutralization plant installed at the University of California at Santa Barbara. This case study has become a standard for comparing single loop control strategies (Hu, Saha, & Rangaiah, 2000; Townsend et al., 1998; Lightbody, O'Reilly, Irwin, Kelly, & McCormick, 1997; Nahas, Henson, & Seborg, 1992).

The process consists of acid, base and buffer stream being continually mixed in a vessel. The control objective is to control the value of the pH of the outlet stream, Q_4 , by varying the inlet base flow rate, Q_2 . The acid and buffer flow rates, Q_1 and Q_3 , respectively, are controlled using peristaltic pumps. The outlet flow rate is dependent on the fluid height in the vessel and the position of the manual outlet valve. The pH of the outlet stream is measured at a distance from the plant, which introduces a measurement time delay, θ .

The process model is derived by defining reaction invariants as (Nahas et al., 1992)

$$W_a \Delta [H^+] - [OH^-] - [HCO_3^-] - 2 \times [CO_3^{2-}], \quad (38)$$

$$W_b \Delta [H_2CO_3] + [HCO_3^-] + [CO_4^{2-}]. \quad (39)$$

Eq. (38) represents a charge balance while Eq. (39) describes the balance on the carbonate ion. Unlike the pH, the reaction invariants are conserved. The dynamic process model consists of three nonlinear ordinary differential equations and a nonlinear output equation for the pH:

$$\dot{h} = \frac{1}{A_r} (Q_1 + Q_2 + Q_3 - C_v h^{0.5}), \quad (40)$$

$$\dot{W}_a = \frac{1}{A_r h} [(W_{a1} - W_{a4})Q_1 + (W_{a2} - W_{a4})Q_2 + (W_{a3} - W_{a4})Q_3], \quad (41)$$

$$\dot{W}_b = \frac{1}{A_r h} [(W_{b1} - W_{b4})Q_1 + (W_{b2} - W_{b4})Q_2 + (W_{b3} - W_{b4})Q_3], \quad (42)$$

$$W_{a4} + 10^{pH_4 - 14} + W_{b4} \frac{1 + 2 \times 10^{pH_4 - pK_2}}{1 + 10^{pK_1 - pH_4} + 10^{pK_2 - pH_4}} - 10^{-pH_4} = 0. \quad (43)$$

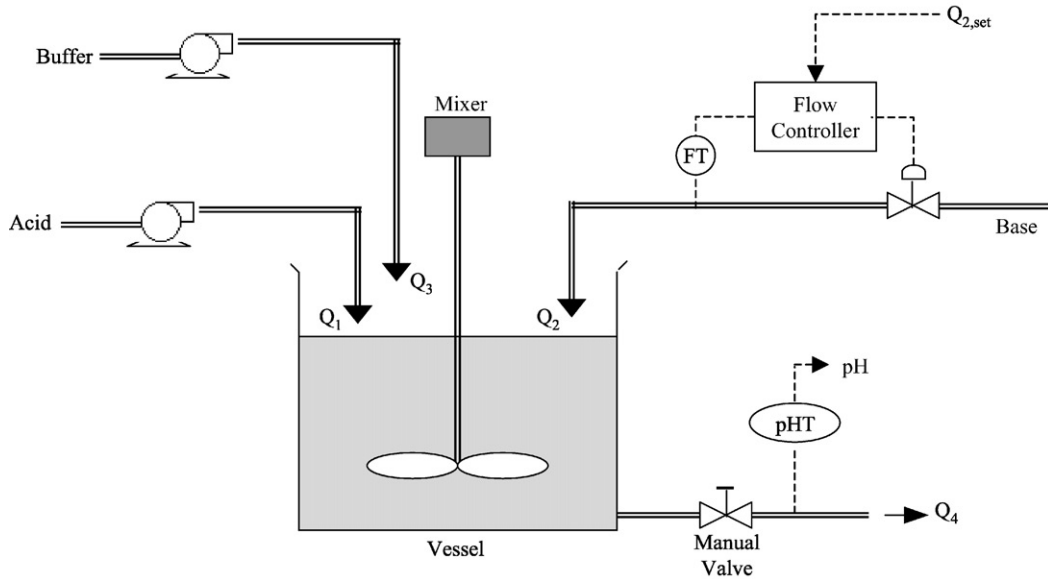


Fig. 5. pH Neutralization Plant Graphic.

The initial model parameters and operating conditions are given in Table 5. Randomly distributed white noise was added to the simulation. Further details for the model and operating conditions can be found in Hu et al. (2000), Townsend et al. (1998), Lightbody et al. (1997), and Nahas et al. (1992).

Three sets of test data were obtained at pH (measured process variables) of 3.9, 7.6, and 10.5. Dynamic tests are performed by pulsing the controller output at each level of operation, generating three sets of test data. Following the procedure, each data set is fit with a FOPDT model (results listed in Table 6) and these parameters are used to compute the adaptive DMC tuning values (results listed in Table 7).

For the set point tracking capabilities, the pH was initially set to a value of 4.0. Then the set point of the pH was stepped by a value of 1.0 until the set point reached a pH value of 9.0. This was done to move the pH process through a wide operating space in which the process gain varies. The design goal for the study is a quick rise time with a 5% POR.

Non-adaptive DMC employs the tuning parameters associated with the middle level of operation (i.e. the measured process variable equals 7.6). Fig. 6 displays the response of the process variable for both the non-adaptive and adaptive DMC implementations. For the set point step changes from a pH value of 4–5 and 7–8, the response of the process variable for the non-adaptive DMC controller shows a POR of 20% and a POR of 50% for the set point step from 7 to 8.

The adaptive DMC controller, on the other hand, was able to maintain the design goal of a quick rise time and a 5% POR over most of the operating range. For the set point step from a value of 7–8, the response for the

Table 5
Nominal pH system operating conditions

$A = 207 \text{ cm}^2$	$W_{b2} = 3 \times 10^{-2} \text{ M}$
$C_v = 8.75 \text{ ml cm}^{-1} \text{ s}^{-1}$	$W_{b3} = 5 \times 10^{-5} \text{ M}$
$pK_1 = 6.35$	$\theta = 0.5 \text{ min}$
$PK_2 = 10.25$	$Q_1 = 16.6 \text{ ml s}^{-1}$
$W_{a1} = 3 \times 10^{-3} \text{ M}$	$Q_2 = 0.55 \text{ ml s}^{-1}$
$W_{a2} = -3 \times 10^{-2} \text{ M}$	$Q_3 = 15.6 \text{ ml s}^{-1}$
$W_{a3} = -3.05 \times 10^{-3} \text{ M}$	$h = 14.0 \text{ cm}$
$W_{b1} = 0$	$\text{pH}_4 = 7.0$

Table 6
FOPDT parameters and DMC tuning parameters for the pH neutralization system

	Lower level	Middle level	Upper level
Process variable value (pH)	3.9	7.6	10.5
FOPDT model parameters			
K_p (pH ml ⁻¹ s ⁻¹)	0.88	0.99	0.06
τ_p (min)	0.95	0.57	1.63
θ_p (min)	0.67	0.5	0.69
DMC tuning parameters			
T (s)		12	
P (samples)		17	
N (samples)		17	
M (samples)		5	
λ		4.76	

adaptive DMC controller exhibits a 15% POR. The adaptive controller was unable to maintain the design goal at this level of operation because of the highly nonlinear process dynamics. In order for the adaptive

Table 7
Adaptive DMC tuning parameters for the pH neutralization system

	Lower level	Middle level	Upper level
Process variable value (pH)	3.9	7.6	10.5
T (s)	12	12	12
P (samples)	41	41	41
N (samples)	41	41	41
M (samples)	10	10	10
λ	21.1	30.5	0.067

controller to maintain a more consistent performance at each level of operation, more linear non-adaptive controllers should be designed and weighted.

As evidenced by the figure, the adaptive DMC controller is able to maintain better performance over all operating ranges than the non-adaptive DMC controller. This example demonstrates the feasibility of the adaptive DMC algorithm for a highly nonlinear process simulation that is representative of an experimental pilot plant.

4.4. Gravity drained tanks experiment

A schematic of the experimental gravity drained tanks unit installed at the University of Connecticut is shown

in Fig. 7. This experimental system consists of two non-interacting tanks stacked one above the other. The two tanks are each of 3 in diameter and 24 in height. Liquid drains freely through a hole in the bottom of each tank. The bottom tank drains into a bucket that collects the water and serves as a reservoir for the pump. The small variable speed pump is used to pump the water from the reservoir into the upper tank. The objective of the control system is to maintain the liquid level in the bottom tank by controlling the amount of water fed to the upper tank.

The controller output manipulates the inlet flow rate into the top tank. The measured process variable is the liquid level of the bottom tank. This level is measured using a differential pressure sensor. The process displays a nonlinear behavior in that the process gain changes by a factor of 3, the overall process time constant changes by a factor of 2.5, and the overall dead time changes by a factor of 2 over the range studied in this example.

Three sets of test data were obtained at lower tank levels (measured process variables) of 1, 4, and 8 in. Dynamic tests are performed by pulsing the controller output at each level of operation, generating three sets of test data. Process models were developed from this test data. As in the previous example, each data set is fit

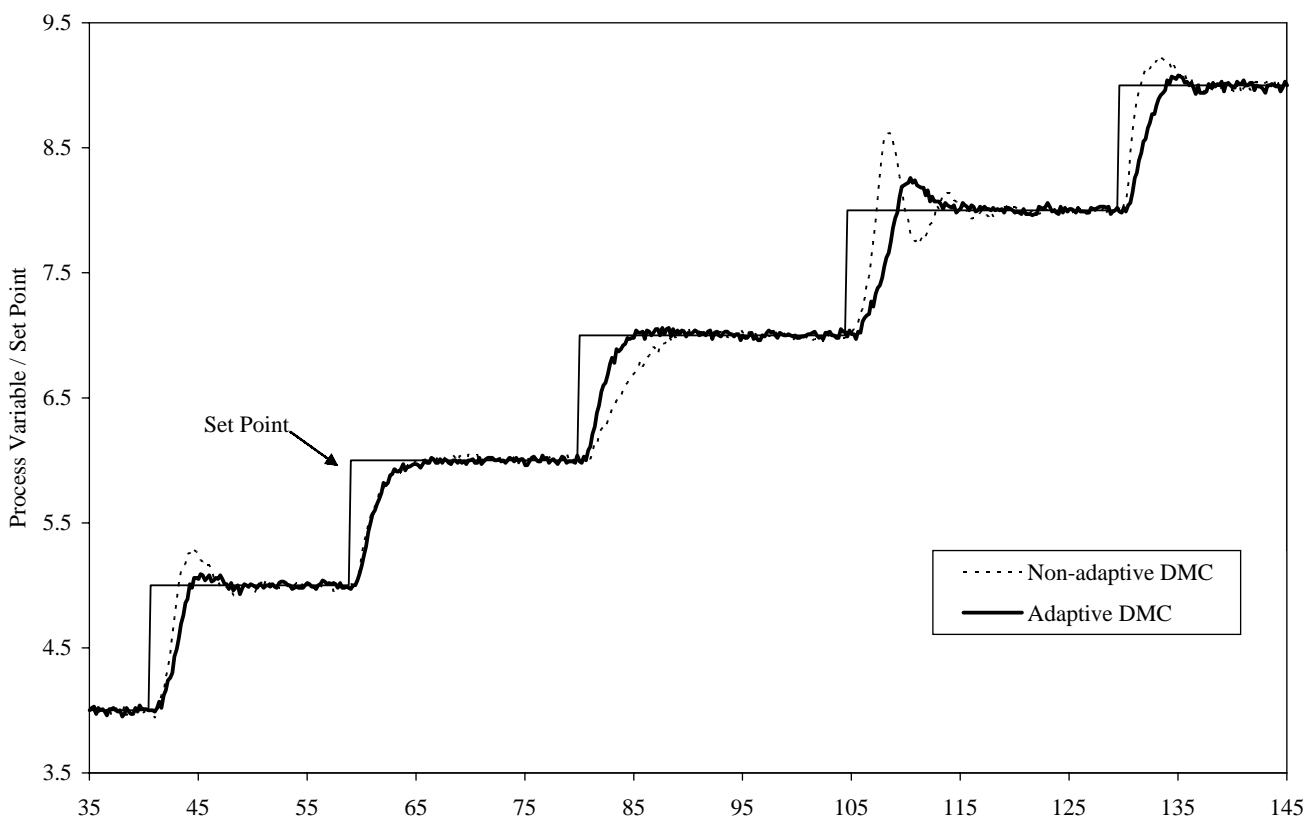


Fig. 6. Response of the process variable for the pH neutralization system using non-adaptive and adaptive DMC for set point tracking.

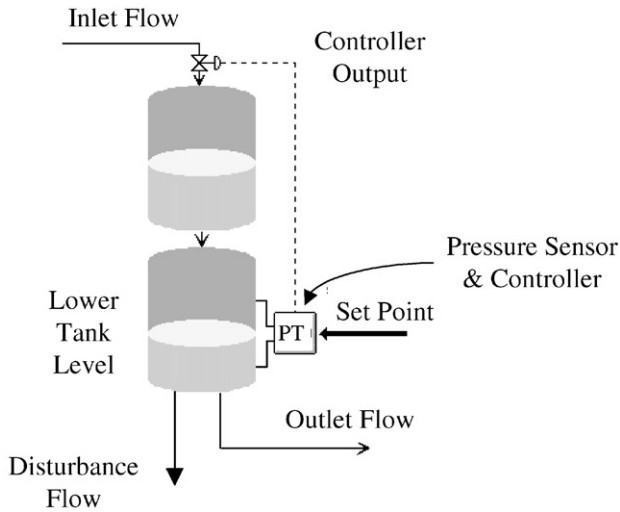


Fig. 7. Gravity drained tanks experiment graphic.

Table 8
FOPDT parameters and DMC tuning parameters for the gravity drained tanks experiment

	Lower level	Middle level	Upper level
Process variable value (in)	1.0	4.0	8.0
FOPDT model parameters			
K_p (in/%)	0.061	0.12	0.17
τ_p (min)	0.77	1.4	1.93
θ_p (min)	0.50	0.75	0.94
DMC tuning parameters			
T (s)		24	
P (samples)		19	
N (samples)		19	
M (samples)		5	
λ		0.088	

Table 9
Adaptive DMC tuning parameters for the gravity drained tanks experiment

	Lower level	Middle level	Upper level
Process variable value (in)	1.0	4.0	8.0
T (s)	12	12	12
P (samples)	53	53	53
N (samples)	53	53	53
M (samples)	14	14	14
λ	0.045	0.40	1.2

with a FOPDT model (results listed in Table 8) and these parameters are used to compute the adaptive DMC tuning values (results listed in Table 9).

Process constraints were included so as to compare non-adaptive and adaptive QDMC. The constraints

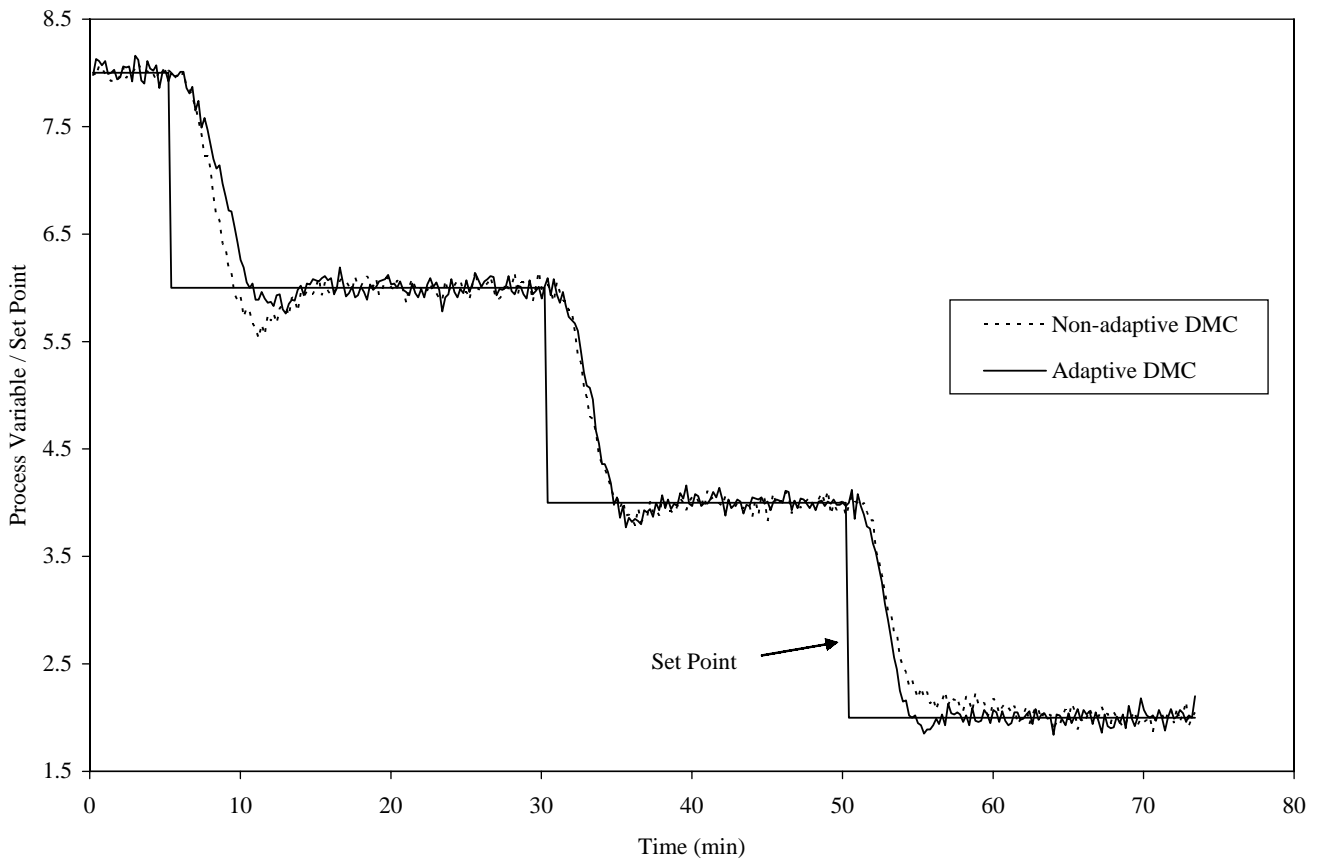


Fig. 8. Response of the process variable for the gravity drained tanks experiment using non-adaptive and adaptive QDMC for set point tracking.

considered in this investigation include:

$$1 \leq \hat{y} \leq 8, \quad (44a)$$

$$-2 \leq \Delta \bar{u} \leq 2, \quad (44b)$$

$$0 \leq \bar{u} \leq 100. \quad (44c)$$

Non-adaptive QDMC employs the tuning parameters associated with the middle level of operation (i.e. the measured process variable equals 4 in). Fig. 8 displays the response of the process variable for both the non-adaptive and adaptive QDMC implementations. The design goal for the study is a quick rise time with a 10% POR.

The response of the process variable for the non-adaptive QDMC controller displays a 25% POR for the set point step from 8 to 6 in. For the set point step from 4 to 2 in, the non-adaptive controller exhibits a sluggish response with no POR. The adaptive QDMC controller is able to maintain the set point tracking design goals over the entire range of operation.

The disturbance rejection capabilities of the adaptive and non-adaptive QDMC controller were also studied. The disturbance is a secondary flow out of the lower tank from a positive displacement pump, and is

independent of the liquid level except when the tank is empty. The disturbance flow rate was stepped from 0 to 2 ml min^{-1} and then back to 0 ml min^{-1} .

Fig. 9 shows the response of the process variables for both the non-adaptive and adaptive QDMC implementations at a set point level of 4 in. At this level of operation both the adaptive and non-adaptive QDMC controllers give similar performance. This is because the non-adaptive controller was designed for a level of 4 in. This is verified in Fig. 9.

Fig. 10 displays the response of the process variables for both the non-adaptive and adaptive QDMC implementations at a set point level of 1 in. At this level of operation the adaptive QDMC controller should exhibit better disturbance rejection capabilities. This is because the tuning and model parameters for the non-adaptive controller are no longer valid. As displayed in Fig. 10, the adaptive controller outperforms the non-adaptive controller. The adaptive controller is able to reject the disturbance quicker and return the height of the tank back to its set point faster. In addition, the response of the process variable for the adaptive DMC controller exhibits a smaller overshoot ratio.

As shown by these figures, the adaptive QDMC controller is able to maintain better performance over all

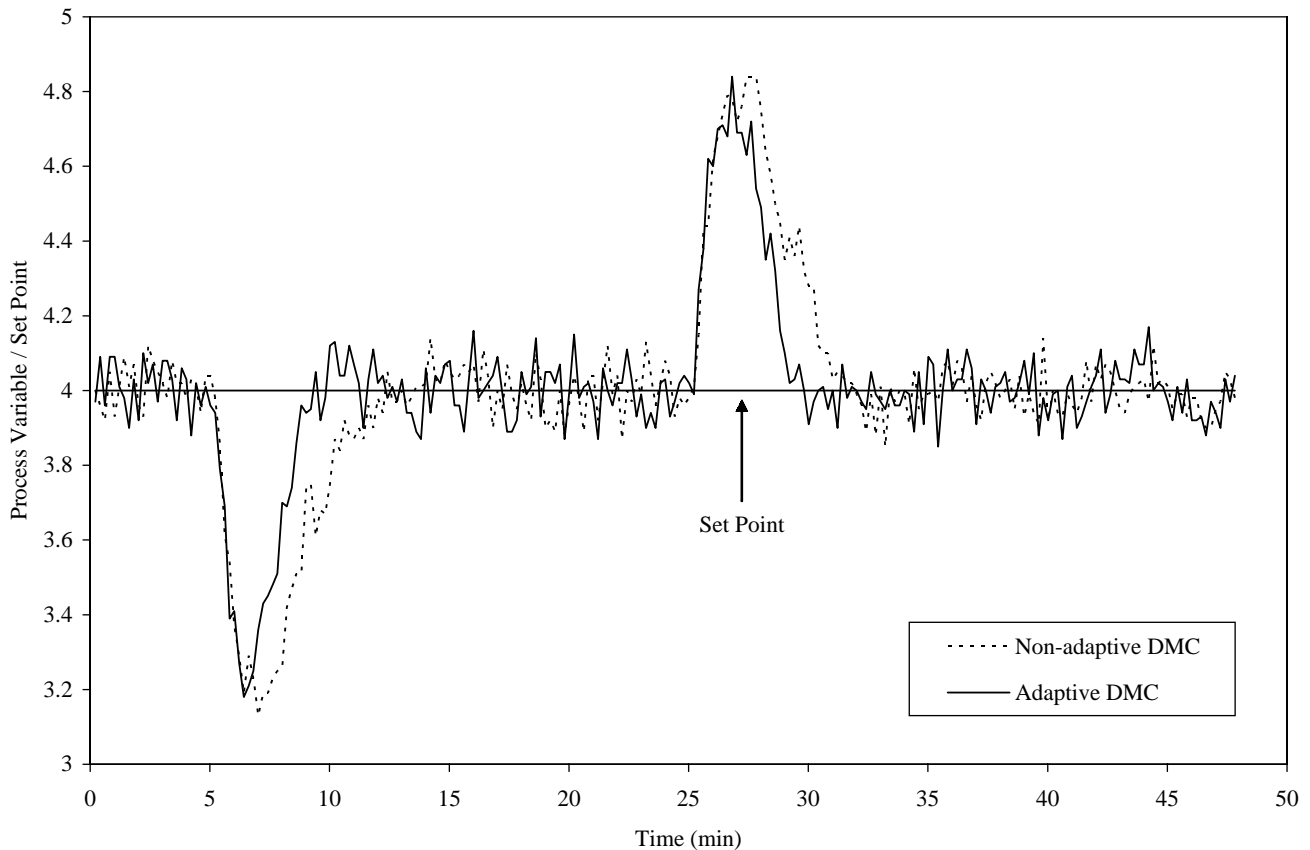


Fig. 9. Response of the process variable for the gravity drained tanks experiment using non-adaptive and adaptive QDMC for disturbance rejection capabilities at a set point of 4 in.

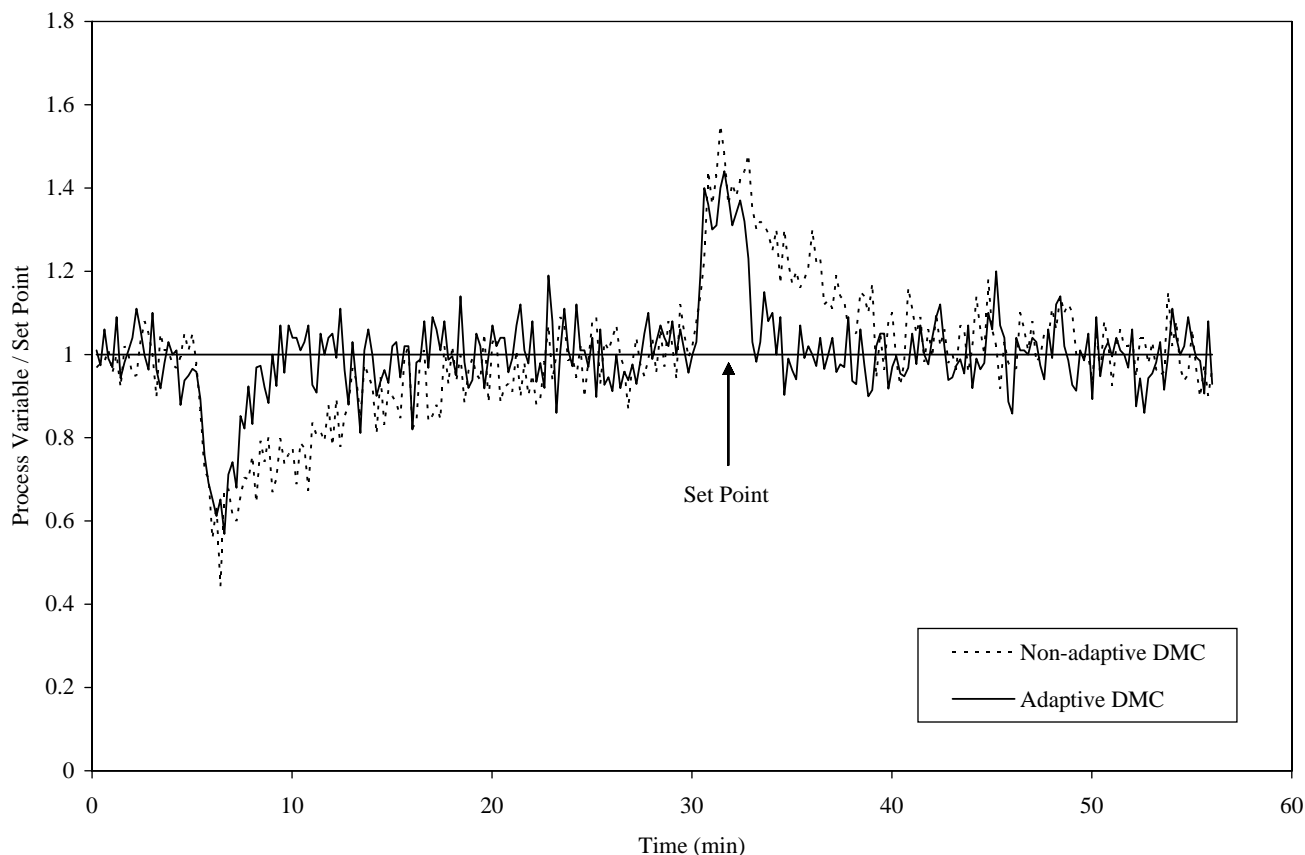


Fig. 10. Response of the process variable for the gravity drained tanks experiment using non-adaptive and adaptive QDMC for disturbance rejection capabilities at a set point of 1 in.

operating ranges. The adaptive strategy weights the multiple controller output moves in order to achieve the desired performance at each level of operation.

5. Conclusions

A multiple model adaptive strategy for single-loop DMC and QDMC is presented. The application and benefits of this adaptive strategy is demonstrated through simulation examples and a practical laboratory application. For the non-adaptive DMC algorithm, the process variable responses varied greatly from over-damped to under-damped depending on the operating level. However, the adaptive DMC controller is able to maintain better set point tracking performance and disturbance rejection capabilities over the range of nonlinear operation. This work develops an adaptive strategy that builds upon linear controller design methods for creating a robust MMAC for DMC and QDMC. The contributions of the method presented here include an adaptive DMC strategy that:

- is straightforward to implement and use,
- requires minimal computation for updating model parameters,

- relies on the linear control knowledge of plant personnel, and
- is reliable for a broad class of process applications.

The development of a multiple model adaptive strategy for multiple-input multiple-output (MIMO) DMC is critical to the practitioner. In many industrial applications, when one controller output variable is changed it will not only affect the corresponding measured process variable, but it also will have an impact on the other measured process variables. The MMAC algorithm for single-loop DMC provides the foundation upon which a multiple model algorithm can be developed for multivariable DMC.

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