Automatica 89 (2018) 333–339

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Robust self-triggered min–max model predictive control for discrete-time nonlinear systems[☆]

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ARTICLE INFO

Article history:
Received 24 December 2016
Received in revised form 20 November 2017
Accepted 27 November 2017

Keywords:
Nonlinear systems
Self-triggered control
Min–max model predictive control
Robust control

ABSTRACT

In this paper, we propose a robust self-triggered model predictive control (MPC) algorithm for constrained discrete-time nonlinear systems subject to parametric uncertainties and disturbances. To fulfill robust constraint satisfaction, we take advantage of the min–max MPC framework to consider the worst case of all possible uncertainty realizations. In this framework, a novel cost function is designed based on which a self-triggered strategy is introduced via optimization. The conditions on ensuring algorithm feasibility and closed-loop stability are developed. In particular, we show that the closed-loop system is input-to-state practical stable (ISpS) in the attraction region at triggering time instants. In addition, we show that the main feasibility and stability conditions reduce to a linear matrix inequality for linear case. Finally, numerical simulations and comparison studies are performed to verify the proposed control strategy.

1. Introduction

In modern cyber-physical systems (CPSs) where control inputs are generally transmitted via shared communication networks, there is a desire to balance the closed-loop control performance with the communication cost necessary to achieve this performance. In conventional time-triggered control, the measurement signal is sampled with a fixed time period, and the control signal is calculated and implemented periodically without considering dynamical characteristics of the system. This may result in greater utilization of computation and communication resources than it necessarily needs. To achieve a better trade-off between control performance and communication cost in CPSs, event-triggered control has been proposed in the literature, e.g. Heemels, Johansson, and Tabuada (2012) and Lucia, Kögel, Zometa, Quevedo, and Findeisen (2016). It determines online when to communicate and actuate by monitoring closed-loop system behaviors, leading to possible aperiodic control with smaller average sampling rate.

Several results addressing aperiodic control have been reported in the literature. For example, the controllers with aperiodic scheduling of input execution have been examined in Eqtami, Dimarogonas, and Kyriakopoulos (2010) and Tabuada (2007) for undisturbed systems, and in Donkers and Heemels (2012), Henningsson, Johannesson, and Cervin (2008), Lunze and Lehmann (2010) and Wang and Lemmon (2009) for disturbed systems. Broadly speaking, these methods can be categorized into either "event-triggered" or "self-triggered" schemes (Heemels et al., 2012). Specifically, in event-triggered control, input signals are computed and applied only when the error between actual and predicted system states deviates away from a prescribed set, and in self-triggered control the next triggering time is pre-computed based on the knowledge of current system state and system dynamics. For a recent overview about control systems with aperiodic sampling, the interested reader is referred to Hetel et al. (2017).

It is well known that model predictive control (MPC) is currently widely utilized in the industrial control systems and has greatly increased profits in comparison with PID control. As communication and networks play more and more important roles in modern society, there is a great trend to upgrade and transform traditional industrial systems into CPSs, which naturally requires extending conventional MPC to communication-efficient MPC to save network resources. In this context, event-triggered MPC comes into being and has received increasing attention recently.

In the literature, event-triggered MPC has been proposed for undisturbed systems (Eqtami et al., 2010; Eqtami, Dimarogonas, & Kyriakopoulos, 2011) and systems with additive disturbances (Brunner, Heemels, & Allgöwer, 2015; Hashimoto, Adachi, &

☆ This work is supported in part by the National Nature Science Foundation of China (NSFC) under Grants 61473225, 61502395 and 51279164; in part by the Fundamental Research Funds for the Central Universities under Grant 3102016AXXX012; in part by Natural Science Foundation of Shaanxi Province under grant 2017JM6058. The material in this paper was partially presented at the 2017 Chinese Control Conference (CCC), July 26–July 28, 2017, Dalian, China. This paper was recommended for publication in revised form by Associate Editor Riccardo Scattolini under the direction of Editor Ian R. Petersen.

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https://doi.org/10.1016/j.automatica.2017.12.034
0005-1098/

Dimarogonas, 2015, 2017a; He & Shi, 2015; Li & Shi, 2014; Li, Yan, Shi, & Wang, 2015; Liu, Gao, Li, & Xu, 2017), respectively. However, event-triggered MPC generally requires continuously sampling system state and then checking triggering conditions, which is not feasible for practical implementation. To overcome this drawback, self-triggered MPC strategies have been proposed in Aydiner, Brunner, Heemels, and Allgöwer (2015), Barradas Berglind, Gommans, and Heemels (2012), Brunner, Heemels, and Allgöwer (2016), Gommans and Heemels (2015) and Hashimoto, Adachi, and Dimarogonas (2017b), where Barradas Berglind et al. (2012) and Gommans and Heemels (2015) deal with undisturbed systems and Aydiner et al. (2015), Brunner et al. (2016) and Hashimoto et al. (2017b) treat systems subject to disturbances.

Self-triggered MPC for uncertain systems is of particular importance as uncertainties are not avoidable in practice, which is also the focus of this paper. Among the results of aperiodic MPC, Eqtami et al. (2010, 2011), He and Shi (2015) and Li and Shi (2014) use nominal models to formulate the optimization problems, the stability is ensured by exploring the inherent robustness of MPC and the original system constraints are tightened to achieve robust constraint satisfaction. In these cases, the closed-loop stability is usually established by exploiting the system inherent robustness. Unfortunately, this method suffers from very small attraction regions, especially for unstable linear systems and nonlinear systems with relatively large Lipschitz constants, due to the constraint tightening procedure. To enlarge attraction region, the authors in Aydiner et al. (2015) and Brunner et al. (2016) recently investigated the robust self-triggered MPC problem for discrete-time linear systems based on the idea of tube-based MPC (Farina & Scattolini, 2012; Mayne, Seron, & Rakovic, 2005), where a pre-stabilizing linear feedback controller is introduced into the prediction model to attenuate disturbance impacts. In contrast to robust self-triggered MPC using a nominal model, self-triggered MPC with a tube-based strategy has less conservative tightened constraints, therefore offering relatively large regions of attraction.

It is worth noting that the existing results of self-triggered MPC might not be able to handle systems with generic parameter uncertainties, though model uncertainties are almost unavoidable in system modeling. Besides, enlarging the region of attraction is always preferred for MPC design. Motivated by these facts, this paper proposes a robust self-triggered min–max MPC approach to constrained nonlinear systems with both parameter uncertainties and disturbances, leading to an enlarged region of attraction in comparison with Brunner et al. (2016).

The main contributions of this work are two-fold:

- A self-triggered min–max MPC algorithm is designed for generic constrained nonlinear system with both parameter uncertainties and disturbances. The designed algorithm is proved to be recursively feasible and the closed-loop system is input-to-state practical stable (ISpS) at triggering time instants in its region of attraction. Compared with existing self-triggered MPC strategies where nominal models are used for prediction, we take advantage of the worst case of all possible uncertainty realizations in the self-triggered control, ensuring robust constraint satisfaction in presence of parametric uncertainties and external disturbances.
- More specific results are developed for linear systems with parameter uncertainties and external disturbances. In particular, we show that for linear systems with additive disturbances, the approximate closed-loop prediction strategy (Goulart, Kerrigan, & Alamo, 2009; Lazar, Muñoz de la Peña, Heemels & Alamo, 2008; Magni, Raimondo, & Scattolini, 2006; Raimondo, Limon, Lazar, Magni, & Camacho, 2009) can be adopted to facilitate the self-triggered min–max linear MPC design to yield an enlarged attraction region, the feasibility and stability conditions reduce to a linear matrix inequality, which can be solved easily.

The notations adopted in this paper are as follows. Let $\mathbb{R}$, and $\mathbb{N}$ denote by the sets of real and non-negative integers, respectively. $\mathbb{R}^n$ denotes the Cartesian product $\underbrace{\mathbb{R}\times\mathbb{R}\cdots\times\mathbb{R}}_{n}$. We use the notations $\mathbb{R}_{\geq c_1}$ and $\mathbb{R}_{(c_1,c_2]}$ to denote the sets $\{t\in\mathbb{R}|t\geq c_1\}$ and $\{t\in\mathbb{R}|c_1<t\leq c_2\}$, respectively, for some $c_1\in\mathbb{R}$, $c_2\in\mathbb{R}_{\geq c_1}$. The notation $\|\cdot\|$ is used to denote an arbitrary p-norm. Given a matrix S, $S\succ 0$ ($S\prec 0$) means that the matrix is positive (negative) definite. A scalar function $\alpha:\mathbb{R}_{\geq 0}\to\mathbb{R}_{\geq 0}$ is of class $\mathcal{K}$ if it is continuous, positive definite and strictly increasing. It belongs to class $\mathcal{K}_\infty$ if $\alpha\in\mathcal{K}$ and $\alpha(s)\to+\infty$ as $s\to+\infty$. For $m,n\in\mathbb{N}_{>0}$, $I_{m\times m}$ denotes an identity matrix of size m and $0_{m\times n}$ represents an $m\times n$ matrix whose entries are zero.

2. Preliminaries and problem statement

2.1. Preliminaries

Consider the discrete-time perturbed nonlinear system given by

$$x_{t+1}=g(x_t,d_t), \tag{1}$$

where $x_t\in\mathbb{R}^n$, $d_t=[w_t^{\mathrm{T}},v_t^{\mathrm{T}}]^{\mathrm{T}}\in\mathcal{D}\subset\mathbb{R}^d$ are the system state, unknown time-varying model uncertainties, respectively, at discrete time $t\in\mathbb{N}$. More specifically, $w_t\in\mathcal{W}\subset\mathbb{R}^w$ denotes parametric uncertainties and $v_t\in\mathcal{V}\subset\mathbb{R}^v$ stands for additive disturbances. $\mathcal{W}$ and $\mathcal{V}$ are compact sets, and contain the origin in their interiors. $g:\mathbb{R}^n\times\mathbb{R}^d\to\mathbb{R}^n$ is a nonlinear function satisfying $g(0,0)=0$.

Definition 1 (*RPI*)**.** A set Ω is a robust positively invariant (RPI) set for the system (1) if $g(x_t,d_t)\in\Omega$, $\forall x_t\in\Omega$, $d_t\in\mathcal{D}$.

Lemma 1 (*Lazar et al., 2008*)**.** *Given an RPI set $\mathcal{X}$ with $\{0\}\subset\mathcal{X}$ for the system (1), let $V:\mathbb{R}^n\to\mathbb{R}_{\geq 0}$ be a function such that: (1) $\alpha_1(\|x\|)\leq V(x)\leq\alpha_2(\|x\|)+\tau_1$; (2) $V(g(x,d))-V(x)\leq-\alpha_3(\|x\|)+\sigma(\|v\|)+\tau_2$, for all $x\in\mathcal{X}$, $d=[w^{\mathrm{T}},v^{\mathrm{T}}]\in\mathcal{D}$, where $\alpha_1(s)\triangleq as^\lambda$, $\alpha_2(s)\triangleq bs^\lambda$ and $\alpha_3(s)\triangleq cs^\lambda$ with $a,b,c,\tau_1,\tau_2,\lambda\in\mathbb{R}_{>0}$ and $c\leq b$, and σ is a $\mathcal{K}$-function, then the system (1) is ISpS in $\mathcal{X}$ with respect to v.*

2.2. Problem statement

Consider a discrete-time perturbed nonlinear system given by

$$x_{t+1}=f(x_t,u_t,d_t), \tag{2}$$

where $x_t\in\mathbb{R}^n$, $u_t\in\mathbb{R}^m$, $d_t=[w_t^{\mathrm{T}},v_t^{\mathrm{T}}]\in\mathcal{D}\subset\mathbb{R}^d$ are the system state, the control input, unknown, possibly time-varying model uncertainties, respectively, at discrete time $t\in\mathbb{N}$. More specifically, $w_t\in\mathcal{W}\subset\mathbb{R}^w$ represents parametric uncertainties and $v_t\in\mathcal{V}\subset\mathbb{R}^v$ stands for additive disturbances. $f:\mathbb{R}^n\times\mathbb{R}^m\times\mathbb{R}^d\to\mathbb{R}^n$ is a nonlinear function satisfying $f(0,0,0)=0$. It is assumed that the system is subject to state and input constraints given by $x_t\in\mathcal{X}$, $u_t\in\mathcal{U}$, where $\mathcal{X}$ and $\mathcal{U}$ are compact sets containing the origin in their interiors. $\mathcal{W}$ and $\mathcal{V}$ are compact sets, and contain the origin in their interiors. We further assume that the state is available as a measurement at any time instant.

The control objective of this paper is to design a self-triggered MPC strategy to robustly asymptotically stabilize the system (2) while satisfying the system constraints. Let the sequence $\{t_k|k\in\mathbb{N}\}\in\mathbb{N}$ where $t_{k+1}>t_k$ be the time instants when optimization problem needs to be solved. In particular, the control law is of the form

$$u_t=\mu(x_{t_k},t-t_k),\ t\in\mathbb{N}_{[t_k,t_{k+1}-1]},$$

where $\mu : \mathbb{R}^n \times \mathbb{N} \to \mathbb{R}^m$ is a function, and $\{t_k | k \in \mathbb{N}\} \in \mathbb{N}$ are sampling instants that are determined by using a self-triggering scheduler, i.e.

$$t_0 = 0,\ t_{k+1} = t_k + H^*(x_{t_k}),\ k \in \mathbb{N},$$

where $H^* : \mathbb{R}^n \to \mathbb{N}_{\geq 1}$ is a function.

3. Robust self-triggered feedback min–max MPC

3.1. min–max optimization

For a given prediction horizon $N \in \mathbb{N}_{\geq 1}$ and $H \in \mathbb{N}_{[1,N]}$, the cost function at time $t_k \in \mathbb{N}$ is formulated as $J_N^H(x_{t_k}, \mathbf{u}_{t_k,N}, \mathbf{d}_{t_k,N}) \triangleq \sum_{j=0}^{H-1} \frac{1}{\beta} L(x_{j,t_k}, u_{j,t_k}) + \sum_{j=H}^{N-1} L(x_{j,t_k}, u_{j,t_k}) + F(x_{N,t_k})$, where $\beta \in \mathbb{R}_{\geq 1}$ is a fixed constant, x_{j,t_k} denotes the predicted state for system (2) at time $j \in \mathbb{N}_{[0,N-1]}$ initialized at $x = x_{t_k}$ with the control input sequence $\mathbf{u}_{t_k,N} = (u_{0,t_k}, \ldots, u_{N-1,t_k})$ and the disturbance sequence $\mathbf{d}_{t_k,N} = (d_{0,t_k}, \ldots, d_{N-1,t_k})$. We assume that L and F are continuous functions. Specifically, the stage cost is given by $L : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}_{\geq 0}$ with $L(0, 0) = 0$, and the terminal cost is given by $F : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ with $F(0) = 0$. The decision variable $\mathbf{u}_{t_k,N}$ is derived by solving the following min–max optimization problem.

$$\begin{aligned}
V_N^H(x_{t_k}) = & \min_{u_{0,t_k} \in \mathcal{U}, \ldots, u_{H-1,t_k} \in \mathcal{U}} \Bigg\{ \max_{d_{0,t_k} \in \mathcal{D}, \ldots, d_{H-1,t_k} \in \mathcal{D}} \\
& \left\{ \sum_{j=0}^{H-1} \frac{1}{\beta} L(x_{j,t_k}, u_{j,t_k}) + V_{N-H}(x_{H,t_k}) \right\} \text{ such that} \\
& x_{H,t_k} \in \mathcal{X}_{N-H}, \forall d_{0,t_k} \in \mathcal{D}, \ldots, d_{H-1,t_k} \in \mathcal{D} \Bigg\}, \\
\text{s.t.}\ \ & x_{0,t_k} = x_{t_k}, \quad x_{j,t_k} \in \mathcal{X},\ j \in \mathbb{N}_{[0,H-1]}, \\
& x_{j+1,t_k} = f(x_{j,t_k}, u_{j,t_k}, d_{j,t_k}),\ j \in \mathbb{N}_{[0,H-1]},
\end{aligned} \tag{3}$$

where

$$\begin{aligned}
V_i(x_{i,t_k}) = & \min_{u_{i,t_k} \in \mathcal{U}} \Bigg\{ \max_{d_{i,t_k} \in \mathcal{D}} \Big\{ L(x_{i,t_k}, u_{i,t_k}) \\
& + V_{i-1}(f(x_{i,t_k}, u_{i,t_k}, d_{i,t_k})) \Big\} \\
& \text{such that } f(x_{i,t_k}, u_{i,t_k}, d_{i,t_k}) \in \mathcal{X}_{i-1}, \forall d_{i,t_k} \in \mathcal{D} \Bigg\},
\end{aligned}$$

where $i \in \mathbb{N}_{[1,N-H]}$ and $\mathcal{X}_i \subseteq \mathcal{X}$ denotes the set of states that can be robustly controlled into the terminal set $\mathcal{X}_f$ in i steps by using feedback laws. The optimization problem is defined for $i = 1, \ldots, N$ with the boundary conditions: $V_0(x) \triangleq F(x)$, $\mathcal{X}_0 \triangleq \mathcal{X}_f$.

The optimal solution of optimization problem (3) is denoted as $\mathbf{u}^*_{t_k,N} = [u^*_{0,t_k}, \ldots, u^*_{N-1,t_k}]$, and the optimal predicted model uncertainty is written as $\mathbf{d}^*_{t_k,N} = [d^*_{0,t_k}, \ldots, d^*_{N-1,t_k}]$. In the sequel, we particularly denote, for the optimization problem in (3) with $\beta = 1$ and $H = 1$, the cost function by $J_N(x_{t_k}, \mathbf{u}_{t_k,N}, \mathbf{d}_{t_k,N})$, the corresponding optimal cost by $V_N(x_{t_k})$, and the initial feasible region by $\mathcal{X}_N$.

Remark 1. It is worth noting that, we formulate a new cost function $J_N^H(.)$ in min–max optimization in order to design a self-triggered strategy. The solution of optimization problem in (3) is a combination of a sequence of control values $u^*_{j,t_k}, j \in \mathbb{N}_{[0,H-1]}$ (generated by open-loop min–max strategy) and a sequence of control policies $u^*_{j,t_k}, j \in \mathbb{N}_{[H,N-1]}$ (generated by feedback min–max strategy). This configuration is necessarily formulated to facilitate the self-triggered design as the state information is not available to construct feedback laws during triggering time instants in self-triggered control; it will reduce to the conventional one in standard feedback min–max MPC by letting $H = 1$ and $\beta = 1$, and recovers the standard open-loop min–max MPC framework (Lazar et al., 2008; Magni et al., 2006; Raimondo et al., 2009) by setting $H = N$ and $\beta = 1$. Also note that the proposed optimization problem can conveniently incorporate the sparsity of control inputs, $u_{j,t_k} = 0, j \in \mathbb{N}_{[1,H-1]}$ or $u_{j,t_k} = u_{0,t_k}, j \in \mathbb{N}_{[1,H-1]}$ as in Brunner et al. (2016), Barradas Berglind et al., (2012), Gommans and Heemels (2015) and Aydiner et al. (2015), if necessary.

3.2. Self-triggering in optimization

At some sampling time instant $t \in \mathbb{N}$, the control input is defined as

$$u_t^{ST}(x_{t_k}) \triangleq u^*_{t-t_k,t_k},\ t \in \mathbb{N}_{[t_k,t_{k+1}-1]}, \tag{4}$$

where $u^*_{t-t_k,t_k},\ t \in \mathbb{N}_{[t_k,t_{k+1}-1]}$ represents the optimal solution of optimization problem (3). It can be observed that the control input u_t^{ST} is open-loop for $t \in \mathbb{N}_{[t_k+1,t_{k+1}-1]}$ since it only depends on the state at the last sampling time instant t_k. The triggering time instants are determined as follows:

$$\begin{aligned}
& t_{k+1} = t_k + H^*(x_{t_k}), \\
& H^*(x_{t_k}) \triangleq \max\{H \in \mathbb{N}_{[1,H_{\max}]} | V_N^H(x_{t_k}) \leq V_N^1(x_{t_k})\},
\end{aligned} \tag{5}$$

where $H_{\max} \in \mathbb{N}_{[1,N]}$ denotes the maximal length of the open-loop phase. The self-triggered min–max MPC strategy is formulated in Algorithm 1.

Algorithm 1 Self-triggered min–max MPC algorithm

Require: Prediction horizon N; design parameters β and $H_{\max}$.
1: Set $t = t_k = k = 0$;
2: **while** The control action is not stopped **do**
3: Measure the current state x_{t_k} of system (2);
4: Solve the optimization problems in (3) and (5), obtain $\mathbf{u}^*(x_{t_k})$ and $H^*(x_{t_k})$;
5: **while** $t \leq t_k + H^*(x_t) - 1$ **do**
6: Apply $u^*_{t-t_k,t_k}$ to the system;
7: Set $t = t + 1$;
8: **end while**
9: Set $k = k + 1$, $t_k = t$;
10: **end while**

4. Analysis

By applying Algorithm 1 to system (2), the closed-loop system becomes

$$x_{t+1} = f(x_t, u_t^{ST}, d_t), \tag{6a}$$

$$u_t^{ST} = u^*_{t-t_k,t_k},\ t \in \mathbb{N}_{[t_k,t_{k+1}-1]}, \tag{6b}$$

$$t_{k+1} = t_k + H^*(x_{t_k}). \tag{6c}$$

Assumption 1. There exist a function $\kappa_f : \mathbb{R}^n \to \mathbb{R}^m$ with $\kappa_f(0) = 0$, a $\mathcal{K}$-function σ, and $\alpha_l, \alpha_f, \alpha_F, \lambda \in \mathbb{R}_{>0}$ with $\alpha_l \leq \alpha_F$ such that: (1) $\mathcal{X}_f \subseteq \mathcal{X}$ and $0 \in \text{int}(\mathcal{X}_f)$; (2) $\mathcal{X}_f$ is an RPI set for system (2) in closed-loop with $u = \kappa_f(x)$; (3) $L(x, u) \geq \alpha_l \|x\|^\lambda$ for all $x \in \mathcal{X}$ and $u \in \mathcal{U}$; (4) $\alpha_f \|x\|^\lambda \leq F(x) \leq \alpha_F \|x\|^\lambda$ for all $x \in \mathcal{X}_f$; (5) $F(f(x, \kappa_f(x), d)) - F(x) \leq -L(x, \kappa_f(x)) + \overline{\sigma}(\| \upsilon \|)$ for all $x \in \mathcal{X}_f$ and $d \in \mathcal{D}$.

Lemma 2. *For all $x_0 \in \mathcal{X}_f$ and any realization of the disturbances $d_t \in \mathcal{D}$ with $t \in \mathbb{N}$, if Assumption 1 holds for system (2), then*

$$F(x_m) - F(x_0) \leq -\sum_{t=0}^{m-1} (L(x_t, \kappa_f(x_t)) - \overline{\sigma}(\| \upsilon_t \|)), \tag{7}$$

where x_m is derived by applying the local stabilizing law κ_f to system (2), and $m \in \mathbb{N}_{[1,N]}$.

Proof. According to Assumption 1, there exists a feedback law κ_f for system (2) such that

$$F(x_{t+1}) - F(x_t) \leq -L(x_t, \kappa_f(x_t)) + \overline{\sigma}(\| v_t \|), \tag{8}$$

for all $x_t \in \mathcal{X}_f$. Since $\mathcal{X}_f$ is an RPI set for system (2) in closed-loop with κ_f, by summing (8) from $t = 0$ to $t = m - 1$, we obtain the inequality (7). ■

Lemma 3. *For the optimization problem defined in (3),*

$$V_N^1(x_{t_k}) \leq V_N(x_{t_k}). \tag{9}$$

Proof. Without loss of generality, assume the solutions corresponding to $V_N(x_{t_k})$ are $\mathbf{u}^*_{t_k,N} = [u^*_{0,t_k}, \ldots, u^*_{N-1,t_k}]$, $\mathbf{d}^*_{t_k,N} = [d^*_{0,t_k}, \ldots, d^*_{N-1,t_k}]$. Due to optimality, we have

$$V_N^1(x_{t_k}) \leq \max_{\mathbf{d}_{t_k,N}} J_N^1(x_{t_k}, \mathbf{u}^*_{t_k,N}, \mathbf{d}_{t_k,N})$$

$$\leq \max_{\mathbf{d}_{t_k,N}} J_N(x_{t_k}, \mathbf{u}^*_{t_k,N}, \mathbf{d}_{t_k,N}) + \frac{1-\beta}{\beta} L(x_{0,t_k}, u^*_{0,t_k})$$

$$= V_N(x_{t_k}) + \frac{1-\beta}{\beta} L(x_{0,t_k}, u^*_{0,t_k}).$$

Since $L(x_{0,t_k}, u^*_{0,t_k}) \geq 0$ and $\beta \in \mathbb{R}_{\geq 1}$, we can obtain the inequality in (9). ■

Theorem 1. *For the perturbed nonlinear system (2) with $x_0 \in \mathcal{X}_N$, suppose that Assumption 1 holds, then Algorithm 1 is recursively feasible, system (2) in closed-loop with the self-triggered feedback min–max MPC control (4) and (5) is ISpS with respect to v in $\mathcal{X}_N$ at triggering time instants.*

Proof. Please see Appendix.

Remark 2. Note that Theorem 1 investigates the stability of the closed-loop system at triggering time instants. For system states at time instants in between, one can ensure $x_t \in \mathcal{X}$. However, if the states in between are expected in a smaller set, one could tighten the state constraints in (3) to achieve the goal, or if the asymptotic stability of the closed-loop system is desired, one could utilize the dual-mode strategy to satisfy the requirement. From the derivations, we can see that there is a trade-off between the frequency of optimization and the size of the convergence set with respect to the control parameter β. (This argument will be elaborated by means of numerical simulations in the sequel.)

5. The case of linear systems with additive disturbances

Consider the following uncertain linear system

$$x_{t+1} = A(w_t)x_t + B(w_t)u_t + v_t, \tag{10}$$

where the pair $(A(w_t), B(w_t))$ is assumed controllable for all $w_t \in \mathcal{W}$. In this case, the feedback control law can adopt the following linear structure for prediction (Löfberg, 2003):

$$\mathbf{u}_{t_k,N} \triangleq \mathbf{c}_{t_k,N} + \mathbf{M}_N^H \mathbf{v}_{t_k,N}, \tag{11}$$

where $\mathbf{c}_{t_k,N} = [c_{0,t_k}, \ldots, c_{N-1,t_k}]^{\mathrm{T}}$ with $c_{\cdot,t_k} \in \mathbb{R}^m$, $\mathbf{v}_{t_k,N}$ denotes disturbance sequence, and

$$\mathbf{M}_N^H = \begin{bmatrix} 0_{Hm\times n} & 0_{Hm\times n} & \cdots & 0_{Hm\times n} & 0_{Hm\times n} \\ M_{H,0} & \cdots & M_{H,H-1} & 0_{m\times n} & 0_{m\times n} \\ \vdots & \ddots & \ddots & \vdots & 0_{m\times n} \\ M_{N-1,0} & \cdots & \cdots & M_{N-1,N-2} & 0_{m\times n} \end{bmatrix}$$

with $M \in \mathbb{R}^{m\times n}$. Note that the disturbance parameterization min–max MPC introduces conservatism, as the inputs to be optimized are not completely free.

In what follows, we consider a particular case, namely, the system matrices A and B are static and known, which is also the system studied in Brunner et al. (2016).

Corollary 1. *For the perturbed linear system (10) with fixed w_t and $x_0 \in \mathcal{X}_N$, consider the stage cost $L(x,u) = x^{\mathrm{T}}C^{\mathrm{T}}Cx + u^{\mathrm{T}}D^{\mathrm{T}}Du$ with $C^{\mathrm{T}}C \succ 0$, $D^{\mathrm{T}}D \succ 0$, $\overline{\sigma}(\| v \|) = \gamma v^{\mathrm{T}}v$ with $\gamma \in \mathbb{R}_{>0}$, $\kappa_f(x) = Kx$ with K being a matrix, and the terminal cost $F(x) = x^{\mathrm{T}}Px$ with $P \succ 0$. If matrices Q, R, P and K are designed by solving the following optimization problem*

$$\min \gamma$$

$$\text{s.t.} \begin{bmatrix} P & 0_{n\times n} & (P(A+BK))^{\mathrm{T}} & C^{\mathrm{T}} & K^{\mathrm{T}}D^{\mathrm{T}} \\ 0_{n\times n} & \gamma I_{n\times n} & P & 0_{n\times n} & 0_{n\times m} \\ P(A+BK) & P & P & 0_{n\times n} & 0_{n\times m} \\ C & 0_{n\times n} & 0_{n\times n} & I_{n\times n} & 0_{n\times m} \\ DK & 0_{m\times n} & 0_{m\times n} & 0_{m\times n} & I_{m\times m} \end{bmatrix} \succ 0, \tag{12}$$

then Algorithm 1 is recursively feasible, and the system (10) in closed-loop with the self-triggered min–max MPC control (4) and (5) is ISpS with respect to v in $\mathcal{X}_N$.

Proof. Assumption 1-(3) and (4) hold since the quadratic cost is used. By pre- and post-multiplying (12) by $\mathrm{diag}\{I, I, P^{-1}, I, I\}$ and using the Schur complement lead to $((A+BK)x + v)^{\mathrm{T}}P((A+BK)x + v) < x^{\mathrm{T}}Px - x^{\mathrm{T}}Qx - x^{\mathrm{T}}K^{\mathrm{T}}RKx + \gamma v^{\mathrm{T}}v$, implying the satisfaction of Assumption 1-(5). $A + BK$ being stable ensures the existence of set $\mathcal{X}_f$. Therefore, Assumption 1-(1) and (2) hold true. Furthermore, the corresponding RPI set $\mathcal{X}_f$ can be calculated as Raković, Kerrigan, Kouramas, and Mayne (2005). The recursive feasibility of Algorithm 1, stability of the closed-loop system can be analogously analyzed as that in Theorem 1. ■

Remark 3. In comparison with conventional min–max MPC, Algorithm 1 might need less computational load. This is because, though the additional optimization problems (at most $H_{\max}$ quadratic programs) need to be solved at each triggering time instant, the optimization frequency is greatly reduced due to the triggering strategy. Also note that for linear case with quadratic cost, the min–max optimization problem (3) can be solved as the conventional min–max MPC in Goulart et al. (2009) and Löfberg (2003).

6. Simulation and comparisons

Consider the discrete-time nonlinear system (Raimondo et al., 2009) as follows

$$\begin{aligned} x_{t+1}(1) &= x_t(1) + Tx_t(2) \\ x_{t+1}(2) &= -\frac{lT}{m}e^{-x_t(1)}x_t(1) + \frac{m-hT}{m}x_t(2) \\ &\quad + \frac{T}{m}u_t - \frac{T}{m}w_t x_t(2) + \frac{T}{m}v_t, \end{aligned} \tag{13}$$

where the system parameters are given by: $m = 1$ kg; $l = 0.33$ N/m; $h = 1.1$ Ns/m; $T = 0.4$ s. The model uncertainties are limited by $-0.1 \leq w_t \leq 0.1$, $-0.2 \leq v_t \leq 0.4$. The system constraints are set as -4.5 N $\leq u_t \leq 4.5$ N, -2 m $\leq x_t(1) \leq 2$ m. The prediction horizon is chosen as $N = 5$. Set $H_{\max} = 4$. The cost function is set as $L(x,u) = x^{\mathrm{T}}Qx + u^{\mathrm{T}}Ru$ with $Q = \mathrm{diag}(0.64, 0.64)$, $R = 1$. By following the method for deriving min–max MPC parameters developed in Raimondo et al. (2009), the

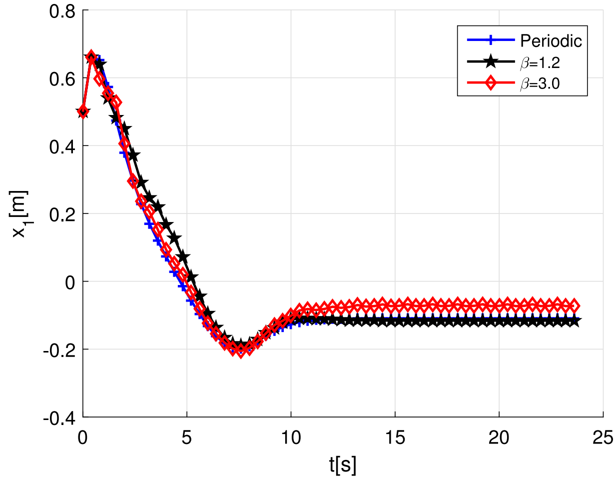

Fig. 1. Trajectories of system state x_1.

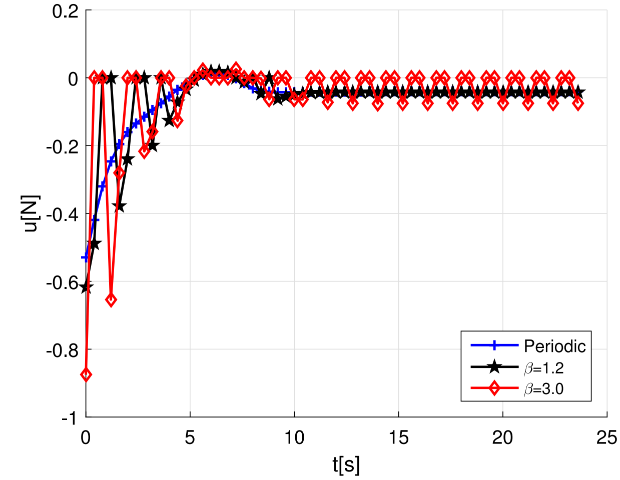

Fig. 3. Trajectories of control input u.

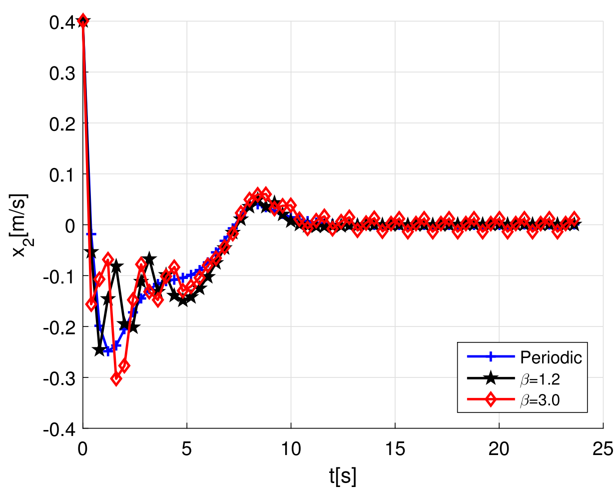

Fig. 2. Trajectories of system state x_2.

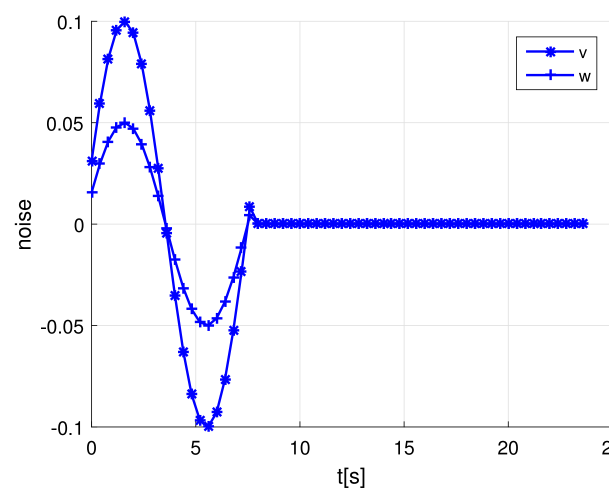

Fig. 4. Trajectories of disturbances.

local stabilizing law and terminal stage cost are derived as $\kappa_f(x) = \begin{bmatrix}-0.7797 & -1.1029\end{bmatrix} x$, $F(x) = x^{\mathrm{T}}Px$ with $P = \begin{bmatrix}4.5678 & 3.2018\\ 3.2018 & 4.3500\end{bmatrix}$, respectively. The terminal region is numerically chosen as $\mathcal{X}_f = \{x : x^{\mathrm{T}}Px \leq 3.8\}$. The policies $u(x) = a\kappa_f(x) + b(x_1^2 + x_2^2) + c$, where $a, b, c \in \mathbb{R}$, are used for prediction from the prediction horizon $N - H$ to N. The initial state is given by $x_0 = [0.5, 0.4]$.

The simulation is conducted by following the self-triggered min–max MPC Algorithm 1, where the MATLAB subroutine `fminimax` is employed to solve constrained min–max optimization problems. We consider two configurations in the simulation, that is, $\beta = 1.2$ and $\beta = 3$. Besides, the periodic min–max robust MPC is also executed in the simulation with the same system parameters. In the simulation, the chosen trajectories of uncertainties are plotted in Fig. 4. The results are reported as follows. Figs. 1–2 show the evolutions of system states, and Fig. 3 depicts the control input. To further illustrate the difference of control performance, the performance indices $J_{\mathrm{p}} = \frac{\sum_{t=0}^{T_{\mathrm{sim}}-1} x_t^{\mathrm{T}}Qx_t + u_t^{\mathrm{T}}Ru_t}{T_{\mathrm{sim}}}$ and the average sampling instants are presented in Table 1, where T_{sim} is the simulation time. It can be observed from Table 1 that the self-triggered min–max MPC strategy reduces the computation load while achieves comparable control performance as the periodic one. It can also be seen that the proposed self-triggered strategy is feasible and the closed-loop system is stable, and with a larger β, the controller has not only a lower optimization frequency but also a larger convergence set.

Table 1
Performance comparison.

	Average sampling time	J_{p}
Periodic	1.0000	0.0477
$\beta = 1.2$	1.2000	0.0519
$\beta = 3.0$	3.3333	0.0560

7. Conclusion

We have studied the robust self-triggered min–max MPC problem for constrained uncertain discrete-time nonlinear systems. A self-triggered control scheduler has been proposed to maximize the inter-sampling time of feedback min–max MPC, and the algorithm feasibility and closed-loop ISpS at triggering time instants have been proved. Numerical simulations and comparison studies have verified the effectiveness and advantages of the proposed results.

Acknowledgments

The authors would like to thank the Associate Editor and the anonymous reviewers for their constructive suggestions that have improved the paper.

Appendix. Proof of Theorem 1

Without loss of generality, we assume that $x_t = x_{t_k} \in \mathcal{X}_N$ and the calculated span of open-loop phase is $H^*(x_{t_k})$ at time t_k. Due to Assumption 1-2, a vector of feedback control polices can be constructed as a feasible solution for the optimization problem (3) at time t_{k+1} as follows

$$(u^*_{H^*(x_{t_k}),t_k}, \cdots, u^*_{N-1,t_k}, \kappa_f(x_{N,t_k}), \cdots, \kappa_f(x_{N+H^*(x_{t_k})-1,t_k})), \tag{A.1}$$

implying that $\mathcal{X}_N$ is an RPI set for system (2) in closed-loop with the proposed self-triggered min–max MPC law. Note that each element of the vector in (A.1) is a feedback law, i.e., its value depends on the actual disturbance realization.

From the definition of the optimization problem (3), for all $x_{t_k} \in \mathcal{X}_N$ we have

$$V_N^{H^*(x_{t_k})}(x_{t_k}) = J_N^{H^*(x_{t_k})}(x_{t_k}, \mathbf{u}^*_{t_k,N}, \mathbf{d}^*_{t_k,N})$$
$$\geq \min_{\mathbf{u}_{t_k,N}} J_N^{H^*(x_{t_k})}(x_{t_k}, \mathbf{u}_{t_k,N}, \mathbf{0}) \geq \frac{\alpha_l}{\beta}\|x_{t_k}\|^\lambda.$$

For all $x_{t_k} \in \mathcal{X}_N$, we consider

$$J^1_{N+1}(x_{t_k}, \tilde{\mathbf{u}}_{t_k,N+1}, \mathbf{d}_{t_k,N+1}) = \big(-F(x_{N,t_k}) + F(x_{N+1,t_k})$$
$$+ L(x_{N,t_k}, \kappa_f(x_{N,t_k}))\big) + J^1_N(x_{t_k}, \mathbf{u}^*_{t_k,N}, \mathbf{d}_{t_k,N}),$$

where $\tilde{\mathbf{u}}_{t_k,N+1} = [\mathbf{u}^*_{t_k,N}, \kappa_f(x_{N,t_k})]$. By application of point 5 of Assumption 1 and sub-optimality of the control input sequence $\tilde{\mathbf{u}}_{t_k,N+H^*(x_{t_k})}$, it follows, for all $x_{t_k} \in \mathcal{X}_N$,

$$V^1_{N+1}(x_{t_k}) \leq \max_{\mathbf{d}_{t_k,N+1}} \big(J^1_{N+1}(x_{t_k}, \tilde{\mathbf{u}}_{t_k,N+1}, \mathbf{d}_{t_k,N+1})\big)$$
$$\leq V^1_N(x_{t_k}) + \max_{v} \overline{\sigma}(\| v \|).$$

Analogously, we have

$$V^1_N(x_{t_k}) \leq V^1_1(x_{t_k}) + (N-1)\max_{v} \overline{\sigma}(\| v \|)$$
$$\leq F(x_{t_k}) + N\max_{v} \overline{\sigma}(\| v \|) + \frac{1-\beta}{\beta} L(x_{0,t_k}, \kappa_f(x_{0,t_k}))$$
$$\leq \alpha_F\|x_{t_k}\|^\lambda + N\max_{v} \overline{\sigma}(\| v \|) \tag{A.2}$$

for all $x_{t_k} \in \mathcal{X}_f$. Recalling the triggering mechanism in (5), it follows $V^H_N(x_{t_k}) \leq \alpha_F\|x_{t_k}\|^\lambda + N\max_v \overline{\sigma}(\| v \|)$, for all $x_{t_k} \in \mathcal{X}_f$. For $x_{t_k} \in \mathcal{X}_N \setminus \mathcal{X}_f$, one can establish the upper bound of $V^H_N(x_{t_k})$ by following the idea in Limon, Alamo, Salas, and Camacho (2006) (Lemma 1) as follows. Define a set $\mathcal{B}_r = \{x \in \mathbb{R}^n | \ \| x \| \leq r\} \subseteq \mathcal{X}_f$, where $r > 0$. Following the compactness of $\mathcal{X}, \mathcal{U}, \mathcal{W}$ and $\mathcal{V}$, there always exists a finite $\bar{J}_N > 0$ such that $V^H_N(x_{t_k}) \leq \bar{J}_N$ for all $x_{t_k} \in \mathcal{X}_N$. Define $\theta = \max(\alpha_F, \frac{\bar{J}_N}{r^\lambda})$. It follows $V^H_N(x_{t_k}) \leq \theta\|x_{t_k}\|^\lambda + N\max_v \overline{\sigma}(\| v \|)$ for all $x_{t_k} \in \mathcal{X}_N$.

From the triggering mechanism in (5), we have

$$V_N^{H^*(x_{t_{k+1}})}(x_{t_{k+1}}) - V_N^{H^*(x_{t_k})}(x_{t_k})$$
$$\leq V^1_N(x_{t_{k+1}}) - V_N^{H^*(x_{t_k})}(x_{t_k})$$
$$\leq V^1_N(x_{t_{k+1}}) - \max_{d_{0,t_k}\in\mathcal{D},\ldots,d_{H-1,t_k}\in\mathcal{D}}$$
$$\Big\{ \sum_{j=0}^{H^*(x_{t_k})-1} \frac{1}{\beta} L(x_{j,t_k}, u^*_{j,t_k}) + V_{N-H^*(x_{t_k})}(x_{H,t_k})\Big\}$$
$$\leq V^1_N(x_{t_{k+1}}) - V_{N-H^*(x_{t_k})}(x_{t_{k+1}})$$
$$- \sum_{j=0}^{H^*(x_{t_k})-1} \frac{1}{\beta} L(x_{t_k+j}, u^*_{j,t_k}), \forall x_{t_k} \in \mathcal{X}_N. \tag{A.3}$$

By using Lemma 2 and an analogous reasoning as in (A.2), one can get

$$V_N(x_{t_{k+1}}) - V_{N-H^*(x_{t_k})}(x_{t_{k+1}}) \leq H^*(x_{t_k})\max_{v} \overline{\sigma}(\| v \|), \tag{A.4}$$

for $x_{t_{k+1}} \in \mathcal{X}_{N-H^*(x_{t_k})}$. Considering Lemma 3 and plugging (A.4) into (A.3), we have

$$V_N^{H^*(x_{t_{k+1}})}(x_{t_{k+1}}) - V_N^{H^*(x_{t_k})}(x_{t_k})$$
$$\leq -\sum_{j=0}^{H^*(x_{t_k})-1} \frac{1}{\beta} L(x_{t_k+j}, u^*_{j,t_k}) + H^*(x_{t_k})\max_{v} \overline{\sigma}(\| v \|)$$
$$\leq -\frac{1}{\beta}\alpha_l\|x_{t_k}\|^\lambda + H^*(x_{t_k})\max_{v} \overline{\sigma}(\| v \|), \forall x_{t_k} \in \mathcal{X}_N.$$

By now, we have shown that $V_N^{H^*(x_{t_k})}(x_{t_k})$ is an ISpS Lyapunov function at triggering time instants. With the aid of Lemma 1, we can conclude that the closed-loop system (6) is ISpS in $\mathcal{X}_N$ with respect to v at triggering time instants. ■

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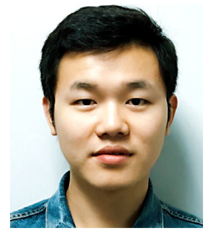

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