Journal of Sound and Vibration (2001) **239**(2), 187–199
doi:10.1006/jsvi.2000.3117, available online at http://www.idealibrary.com on IDEAL®

CONSTRUCTION OF AN ACTIVE SUSPENSION SYSTEM OF A QUARTER CAR MODEL USING THE CONCEPT OF SLIDING MODE CONTROL

T. YOSHIMURA, A. KUME, M. KURIMOTO AND J. HINO

Department of Mechanical Engineering, Faculty of Engineering, The University of Tokushima, Minamijosanjima-cho 2-1, *Tokushima* 770-8506, *Japan. E-mail*: *yosimura@me.tokushima-u.ac.jp*

(*Received* 4 *November* 1999, *and in final form* 23 *March* 2000)

This paper is concerned with the construction of an active suspension system for a quarter car model using the concept of sliding mode control. The active control is derived by the equivalent control and switching function where the sliding surface is obtained by using Linear quadratic control (LQ control) theory. The active control is generated with non-negligible time lag by using a pneumatic actuator, and the road profile is estimated by using the minimum order observer based on a linear system transformed from the exact non-linear system. The experimental result indicates that the proposed active suspension system is more effective in the vibration isolation of the car body than the linear active suspension system based on LQ control theory and the passive suspension system.

1. INTRODUCTION

The investigations of active suspension systems for car models have drawn much interest in recent years [1–3]. The active suspension system is more effective to ride comfort of passengers, but its construction generally provides higher production cost than semi-active and passive suspension systems. The construction of the active suspension system is mainly based on Linear quadratic control (LQ control) theory [4] where a car model is assumed to be a linear or approximate linear system. However, as the car model is practically denoted as a complicated system including non-negligible non-linearity and uncertainty, the derivation of the active control becomes relatively complicated. Recently, various kinds of active suspension systems have been derived for such complicated systems using the concepts of fuzzy reasoning [5], neural network [6] and sliding mode theory [7]. The obtained active suspension systems provide more effective performance in the vibration isolation of the car body, but need more complicated structure in the suspension system than the linear active suspension system derived on the basis of LQ control theory.

The purpose of this paper is to propose an active suspension system of a quarter car model using the concept of sliding mode [8–11]. The sliding mode control denoted as the active control is relatively simpler in the structure of the controller than those based on fuzzy reasoning and neural network, and it guarantees the system stability. The quarter car model to be considered here is approximately described as a non-linear two-degrees-of-freedom (two-d.o.f.) system subject to excitation from a road profile. The time variation of the road profile is assumed to be unknown, and it is estimated by using the minimum order observer on the basis of a linear system transformed from the exact non-linear system.

2. QUARTER CAR MODEL

The experimental apparatus of a quarter car model is shown in Figure 1. The car model is vertically confined by two polls, and m_1 and m_2, respectively, correspond to the masses of the car body and the wheel. The restoring force of the suspension part is constructed by two coil springs with the stiffness k_1, and by four coil springs with the stiffness k_1' as the suspension displacement, $z_1 - z_2$, is larger than a, or less than $-a$. Therefore, the restoring force $f(z_1 - z_2)$ is described by

$$f(z_1 - z_2) = \begin{cases} (k_1 + k_1')(z_1 - z_2) - ak_1' & \text{for } z_1 - z_2 > a, \\ k_1(z_1 - z_2) & \text{for } |z_1 - z_2| \leqslant a, \\ (k_1 + k_1')(z_1 - z_2) + ak_1' & \text{for } z_1 - z_2 < -a. \end{cases} \tag{1}$$

The gravity mainly due to the masses, m_1 and m_2, is supported by the mass m_3 and the coil spring with the stiffness K. The tire stiffness of the wheel is given by the coil spring with the stiffness k_2', and the excitation force generated by the vibrator connected to the signal function generator (SFG) is denoted by f_e. The damping force of the suspension part is assumed equivalently equal to a sum of the Coulomb damping caused by contact with two polls and the viscous damping caused by the pneumatic cylinder. As it is considered relatively small, it is assumed to be linear damping with the damping coefficient c.

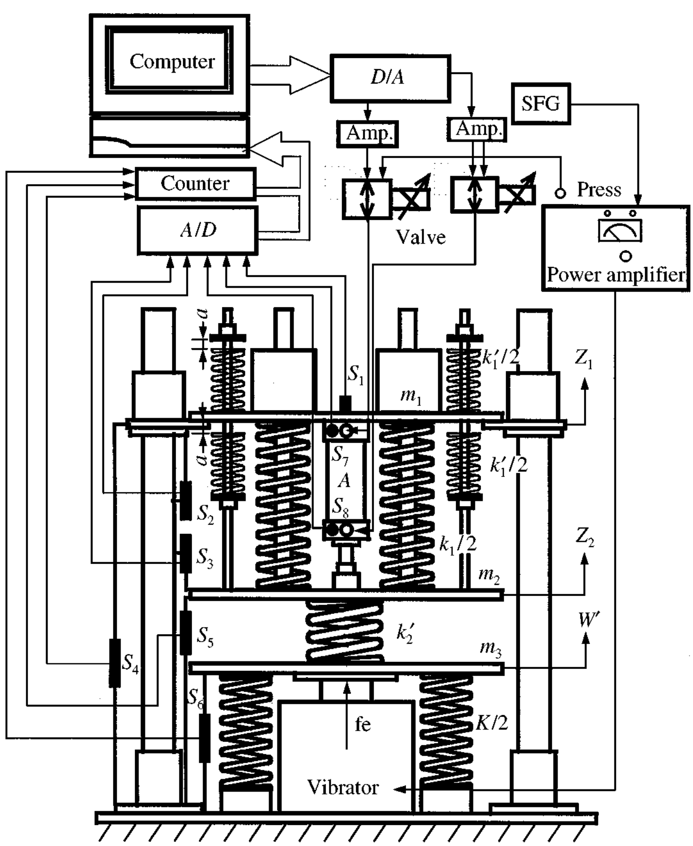

Figure 1. Quarter car model.

Therefore, the equations of motion for the quarter car model are given by

$$m_1\ddot{z}_1 + c(\dot{z}_1 - \dot{z}_2) + f(z_1 - z_2) = u, \tag{2}$$

$$m_2\ddot{z}_2 - c(\dot{z}_1 - \dot{z}_2) - f(z_1 - z_2) + k_2'(z_2 - w') = -u, \tag{3}$$

$$m_3\ddot{w}' + k_2'(w' - z_2) + Kw' = f_e, \tag{4}$$

where u is the active control generated by using a pneumatic actuator. Neglecting the first term on the left-hand side of equation (4) because the mass m_3 is relatively small compared with the masses, m_1 and m_2, defining that

$$k_2 = k_2'K/(k_2' + K), \quad w = f_e/K, \quad z_2 - w = (k_2'/k_2)(z_2 - w'),$$

and substituting the variables defined above into equation (3), then the resultant equation can be expressed as

$$m_2\ddot{z}_2 - c(\dot{z}_1 - \dot{z}_2) - f(z_1 - z_2) + k_2(z_2 - w) = -u, \tag{5}$$

where w and $z_2 - w$ in equation (5) equivalently correspond to the excitation from the road profile and the tire deflection respectively. Therefore, the system described by equations (2) and (5) indicates that the quarter car model is expressed as a non-linear two-d.o.f. system subject to the excitation from the road profile w.

The control part in the experimental apparatus of the quarter car model provides the accelerometer (S_1), the velocity sensors (S_2 and S_3), the linear encoders (S_4, S_5 and S_6), and the pressure gauges (S_7 and S_8). The state variables of the quarter car model can be directly measured by using the sensors, and the control signal is calculated by the personal computer based on the measurement data. The pneumatic control valve is regulated by the control signal through the D/A converter and the power amplifier, and finally the active control u is generated as $u = 77{\cdot}0v$ where v denotes the voltage of the pneumatic control valve. However, as the active control u is generated with non-negligible time delay in the operation of the pneumatic control valve, the algorithm to generate the active control is compensated.

3. CONSTRUCTION OF AN ACTIVE SUSPENSION SYSTEM

Defining the extended state vector $\mathbf{x}$ as

$$\mathbf{x} = [\dot{z}_1 \ z_1 \ \dot{z}_2 \ z_2 \ w]^{\mathrm{T}}$$

the state equation can be written as

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{F}(\mathbf{x}) + \mathbf{b}u + \mathbf{d}\xi(t) \\ &= \mathbf{A}\mathbf{x} + \varDelta\mathbf{F}(\mathbf{x}) + \mathbf{b}u + \mathbf{d}\xi(t),\end{aligned} \tag{6}$$

where $\xi(t)$ is assumed to be a bounded, unknown time-varying function denoted as $\dot{w} = \xi(t)$. On the right-hand side of equation (6), $\mathbf{F}(\mathbf{x})$ and $\varDelta\mathbf{F}(\mathbf{x})$ denote, respectively, the non-linear function of $\mathbf{x}$ and the error due to the linear approximation defined as, $\mathbf{F(x)} - \mathbf{Ax}$ since

$$\mathbf{F(x)} = \begin{bmatrix} -\dfrac{c}{m_1}(\dot{z}_1 - \dot{z}_2) - \dfrac{1}{m_1}f(z_1 - z_2) \\ \dot{z}_1 \\ \dfrac{c}{m_2}(\dot{z}_1 - \dot{z}_2) + \dfrac{1}{m_2}f(z_1 - z_2) - \dfrac{k_2}{m_2}(z_2 - w) \\ \dot{z}_2 \\ 0 \end{bmatrix}, \quad \Delta\mathbf{F(x)} = \begin{bmatrix} -\dfrac{k_1'}{m_1}(z_1 - z_2) \\ 0 \\ \dfrac{k_1'}{m_2}(z_1 - z_2) \\ 0 \\ 0 \end{bmatrix},$$

where the error $\Delta\mathbf{F(x)}$ becomes zero as $|z_1 - z_2| \leqslant a$. The matrix $\mathbf{A}$ and vectors, $\mathbf{b}$ and $\mathbf{d}$, are, respectively, defined as

$$\mathbf{A} = \begin{bmatrix} -\dfrac{c}{m_1} & -\dfrac{k_1}{m_1} & \dfrac{c}{m_1} & \dfrac{k_1}{m_1} & 0 \\ 1 & 0 & 0 & 0 & 0 \\ \dfrac{c}{m_2} & \dfrac{k_1}{m_2} & -\dfrac{c}{m_2} & -\dfrac{k_1 + k_2}{m_2} & \dfrac{k_2}{m_2} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \triangleq \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & 0 \\ 1 & 0 & 0 & 0 & 0 \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{b} = \begin{bmatrix} \dfrac{1}{m_1} & 0 & -\dfrac{1}{m_2} & 0 & 0 \end{bmatrix}^{\mathrm{T}} \triangleq [b_1 \ 0 \ b_3 \ 0 \ 0]^{\mathrm{T}},$$

$$\mathbf{d} = [0 \ 0 \ 0 \ 0 \ 1]^{\mathrm{T}}.$$

Neglecting $\Delta\mathbf{F(x)}$ and $\xi(t)$ in the second line on the right-hand side of equation (6), the state equation becomes

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{b}u. \tag{7}$$

Assuming the performance index,

$$\begin{aligned} J &= E[q_1\dot{z}_1^2 + q_2(z_1 - z_2)^2 + q_3\dot{z}_2^2 + q_4(z_2 - w)^2 + ru^2] \\ &= E[\mathbf{x}^{\mathrm{T}}\mathbf{Qx} + ru^2], \end{aligned} \tag{8}$$

where $q_1 \sim q_4$ and r are positive constants, and the matrix $\mathbf{Q}$ is given by

$$\mathbf{Q} \triangleq \begin{bmatrix} q_1 & 0 & 0 & 0 & 0 \\ 0 & q_2 & 0 & -q_2 & 0 \\ 0 & 0 & q_3 & 0 & 0 \\ 0 & -q_2 & 0 & q_2 + q_4 & -q_4 \\ 0 & 0 & 0 & -q_4 & q_4 \end{bmatrix}.$$

The control that minimizes J with respect to u subject to the constraint of equation (7) is obtained as

$$u_L = -\frac{1}{r}\mathbf{b}^{\mathrm{T}}\mathbf{Px} \triangleq -\mathbf{g}^{\mathrm{T}}\mathbf{x}, \tag{9}$$

where $\mathbf{P}$ is the positive-definite matrix obtained as the solution of the following reduced Ricatti equation ($\dot{\mathbf{P}} = \mathbf{0}$):

$$\mathbf{PA} + \mathbf{A}^{\mathrm{T}}\mathbf{P} - \frac{1}{r}\mathbf{Pbb}^{\mathrm{T}}\mathbf{P} + \mathbf{Q} = \mathbf{0} \tag{10}$$

and $\mathbf{g}$ denotes the control gain. The control u_L is generally called as Linear quadratic control (LQ control). However, u_L does not necessarily minimize the performance index J as it is derived from the linear system transformed from the exact non-linear system.

In order to construct an active suspension system with more improved performance than the linear active suspension system, a sliding mode control is presented as the proposed active control in this paper. Using the control gain $\mathbf{g}$ given by equation (9), the sliding surface s is defined as

$$s = \frac{1}{r}\mathbf{b}^{\mathrm{T}}\mathbf{Px} \triangleq \mathbf{g}^{\mathrm{T}}\mathbf{x} \tag{11}$$

and therefore the equivalent control u_E is derived as [10, 11]

$$u_E = -(\mathbf{g}^{\mathrm{T}}\mathbf{b})^{-1}\mathbf{g}^{\mathrm{T}}\mathbf{F}(\mathbf{x}) \tag{12}$$

by using equation (6) and the following condition:

$$\begin{aligned} \dot{s} &= \mathbf{g}^{\mathrm{T}}\dot{\mathbf{x}} \\ &= \mathbf{g}^{\mathrm{T}}[\mathbf{F}(\mathbf{x}) + \mathbf{b}u_E] \\ &= 0, \end{aligned} \tag{13}$$

where $\xi(t)$ is set to zero as it is unknown. Using the equivalent control u_E, the sliding mode control u_s denoted as

$$u_s = -u_s^*(|u_E| + \varepsilon)\frac{s}{|s|} \tag{14}$$

is proposed where ε is a small positive constant that is added to compensate $u_s \neq 0$ as $|u_E| = 0$, and u_s^* is a positive constant to be determined by the stability condition on the sliding surface s. The stability condition that the system response is restricted on the sliding surface s and slides along the surface to the origin is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}(s^2/2) < 0. \tag{15}$$

The time derivative on the right-hand side of the inequality becomes

$$\begin{aligned} s\dot{s} &= s\mathbf{g}^{\mathrm{T}}[\mathbf{F}(\mathbf{x}) + \mathbf{b}u_s + \mathbf{d}\xi(t)] \\ &= s\mathbf{g}^{\mathrm{T}}[\mathbf{F}(\mathbf{x}) + \mathbf{b}u_E] + s\mathbf{g}^{\mathrm{T}}\mathbf{b}(u_s - u_E) + s\mathbf{g}^{\mathrm{T}}\mathbf{d}\xi(t) \\ &= s\mathbf{g}^{\mathrm{T}}\mathbf{b}(u_s - u_E) + s\mathbf{g}^{\mathrm{T}}\mathbf{d}\xi(t). \end{aligned} \tag{16}$$

Substituting u_s given by equation (14) into the last line on the right-hand side of equation (16), $s\dot{s}$ becomes

$$\begin{aligned} s\dot{s} &= s\mathbf{g}^{\mathrm{T}}\mathbf{b}\left[-u_s^*(|u_E| + \varepsilon)\frac{s}{|s|} - u_E\right] + s\mathbf{g}^{\mathrm{T}}\mathbf{d}\xi(t) \\ &= -u_s^*\mathbf{g}^{\mathrm{T}}\mathbf{b}|s|(|u_E| + \varepsilon) - s\mathbf{g}^{\mathrm{T}}\mathbf{b}u_E + s\mathbf{g}^{\mathrm{T}}\mathbf{d}\xi(t) \\ &\leqslant -u_s^*\mathbf{g}^{\mathrm{T}}\mathbf{b}|s|(|u_E| + \varepsilon) + \mathbf{g}^{\mathrm{T}}\mathbf{b}|s||u_E| + |s||\mathbf{g}^{\mathrm{T}}\mathbf{d}\xi(t)| \end{aligned} \tag{17}$$

as $\mathbf{g}^{\mathrm{T}}\mathbf{b}$ is positive. Therefore, if the value of u_s^* is selected as

$$u_s^* > \frac{|u_E| + \eta}{|u_E| + \varepsilon} \tag{18}$$

with the inequality, $(\mathbf{g}^{\mathrm{T}}\mathbf{b})^{-1}|\mathbf{g}^{\mathrm{T}}\,\mathbf{d}\xi(t)| \leqslant \eta$ $(\eta > \varepsilon)$, then $s\dot{s}$ always becomes negative so that the trajectory of the response is confined to the sliding surface and slides along the surface to the origin. However, equation (18) indicates that the right-hand side becomes smaller (larger) as the value of $|u_E|$ becomes larger (smaller). It means that $\|\mathbf{x}\|$ becomes larger (smaller). That is, the right-hand side of the inequality becomes η/ε as $|u_E| \to 0$ $(\|\mathbf{x}\| \to 0)$ so that u_s^* must be selected as relatively large as η/ε is relatively large. As the amplitude of the active control is bounded in the active suspension system, from the practical viewpoints it is difficult and unnecessary to select a relatively large value of u_s^* as $\|\mathbf{x}\|$ tends to zero.

In this paper, the value of u_s^* therefore is derived by minimizing the performance index J given by equation (8) on the condition that it is a value slightly larger than unity so as to satisfy the stability condition on the sliding surface for a large value of $|u_E|$. However, for the value of u_s^*, if the trajectory of the response is confined to the sliding surface and slides along the surface to the origin for a large value of $|u_E|$, it will separate from the sliding surface and not converge to the origin as the value of $|u_E|$ decreases.

4. CONSTRUCTION OF MINIMUM ORDER OBSERVER

Assuming that $\dot{z}_1$, z_1, $\dot{z}_2$, z_2 are observable and w is unobservable in the state vector $\mathbf{x}$, neglecting the error $\mathbf{\Delta F(x)}$, defining the state vector as

$$\mathbf{x}_1 \triangleq [\dot{z}_1, z_1, \dot{z}_2, z_2]^{\mathrm{T}}$$

and dividing the system given by equation (6) into two kinds of systems, then

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \mathbf{A}_{11}\mathbf{x}_1 + \mathbf{A}_{12}w + \mathbf{b}_1 u, \\ \dot{w} &= \xi(t). \end{aligned} \tag{19}$$

Transforming

$$w^* = w - \mathbf{T}\mathbf{x}_1, \tag{20}$$

where $\mathbf{T} = (T_1, T_2, T_3, T_4)$, the minimum order observer to obtain the estimated value $\hat{w}^*$ of w^* is derived as [12]

$$\dot{\hat{w}}^* = -\mathbf{T}\mathbf{A}_{12}\hat{w}^* - (\mathbf{T}\mathbf{A}_{11} + \mathbf{T}\mathbf{A}_{12}\mathbf{T})\mathbf{x}_1 - \mathbf{T}\mathbf{b}_1 u_s \tag{21}$$

and the estimated value $\hat{w}$ of w is obtained as

$$\hat{w} = \hat{w}^* + \mathbf{T}\mathbf{x}_1. \tag{22}$$

The coefficients characterizing equation (21) are, respectively, given as

$$\mathbf{T}\mathbf{A}_{12} = T_3 A_{35},$$

$$\begin{aligned} \mathbf{T}\mathbf{A}_{11} + \mathbf{T}\mathbf{A}_{12}\mathbf{T} = (&T_1 A_{11} + T_2 + T_3 A_{31} + T_1 T_3 A_{35},\ T_1 A_{12} + T_3 A_{32} + T_2 T_3 A_{35},\ T_1 A_{13} \\ &+ T_3 A_{33} + T_4 + T_3^2 A_{35},\ T_1 A_{14} + T_3 A_{34} + T_3 T_4 A_{35}), \end{aligned}$$

$$\mathbf{T}\mathbf{b}_1 = T_1 b_1 + T_3 b_3.$$

The value of T_3 is taken as positive from the numerical stability in equation (21), and the values of T_1, T_2 and T_4 are free for the selection. The parameters characterizing $\mathbf{T}$ are, respectively, determined so as to minimize the estimation error by performing the experiment.

5. ALGORITHM OF THE ACTIVE CONTROL

As the active control u is generated with non-negligible time delay by the pneumatic actuator operating the pneumatic control valve, the state equation given by equation (6) is rewritten as

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{F}(\mathbf{x}) + \mathbf{b}u(t-d) + \mathbf{d}\xi(t) \\ &= \mathbf{A}\mathbf{x} + \Delta\mathbf{F}(\mathbf{x}) + \mathbf{b}u(t-d) + \mathbf{d}\xi(t),\end{aligned} \tag{23}$$

where d denotes the time delay. As w is replaced by the estimated value $\hat{w}$ in the proposed sliding mode control, then the state vector $\mathbf{x}$ is replaced by $\hat{\mathbf{x}}$ as

$$\hat{\mathbf{x}} = [\dot{z}_1 \; z_1 \; \dot{z}_2 \; z_2 \; \hat{w}]^{\mathrm{T}}.$$

Considering the time delay of the actuator operating the pneumatic control valve, the active control $u_s(t-d)$ becomes

$$u_s(t-d) = -u_s^*[|u_E(\hat{\mathbf{x}}(t))| + \varepsilon]\frac{\mathbf{g}^{\mathrm{T}}\hat{\mathbf{x}}(t)}{|\mathbf{g}^{\mathrm{T}}\hat{\mathbf{x}}(t)|}, \tag{24}$$

where $\hat{\mathbf{x}}(t)$ denotes the predicted value based on the information till $t-d$. It is derived from the relation

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{F}(\hat{\mathbf{x}}(t)) + \mathbf{b}u_s(t-d), \tag{25}$$

where the mean of $\xi(t)$ is assumed to be zero. In order to obtain $\hat{\mathbf{x}}(t)$ in an explicit form, $\Delta\mathbf{F}(\mathbf{x})$ is approximately set to zero as the suspension displacement, $z_1 - z_2$, due to the proposed active control almost lies in the region, $|z_1 - z_2| \leqslant a$. Therefore, $\hat{\mathbf{x}}(t)$ becomes

$$\hat{\mathbf{x}}(t) = \Phi(t)\hat{\mathbf{x}}(t-d) + \int_{t-d}^{t} \Phi(t-\tau)\mathbf{b}u_s(\tau-d)\,\mathrm{d}\tau \tag{26}$$

and $\mathbf{\Phi}(t)$ is given by

$$\mathbf{\Phi}(t) \triangleq £^{-1}[(\lambda\mathbf{I} - \mathbf{A})^{-1}]$$

and $£^{-1}$ denotes the inverse Laplace transformation.

As the active control u_s is generated at the sampling instant with the time interval Δt, equation (26) is approximately expressed as the discrete system

$$\hat{\mathbf{x}}(i) = \Phi(l)\hat{\mathbf{x}}(i-l) + \sum_{j=i-l}^{i-1} \Phi(i-j)\mathbf{b}u_s(j-l)\Delta t, \tag{27}$$

where $t = i\Delta t$, $d = l\Delta t$ and $\tau = j\Delta t$. Therefore, substituting $\hat{\mathbf{x}}(i)$ given by equation (27) into the right-hand side of equation (24), $u_s(i-l)$ is generated by the information of the state $\hat{\mathbf{x}}(i-l)$ and the past active controls $u_s(j-l)\; j = i-l \sim i-1$.

6. RESULT AND DISCUSSION

The parameters of the experimental model are, respectively, given by

$m_1 = 44{\cdot}3$ kg, $m_2 = 11{\cdot}41$ kg, $m_3 = 2{\cdot}88$ kg, $c = 100$ N s/m, $k_1 = 15$ kN/m,

$k_1' = 20$ kN/m, $k_2' = 120$ kN/m, $K = 100$ kN/m, $a = 1{\cdot}4$ mm,

$\Delta t = 0{\cdot}01s$, $d = 0{\cdot}04s$, $l = 4$,

where c and d are, respectively, determined by performing the experiment.

Two kinds of road profiles are considered in the experiment. The first is the sinusoidal road with the frequency 4 Hz, and the second is the random road with the bandwidth 5 Hz. The selection of the frequencies is related to the natural frequency of the car body (about 3 Hz) and the specification of the vibrator and the signal function generator. The weighting factors of the performance index J given by equation (8) are assumed to be

$$q_1 = 1, \quad q_2 = 0{\cdot}1, \quad q_3 = 0{\cdot}01, \quad q_4 = 0{\cdot}1, \quad r = 10^{-8}.$$

As the performance of the minimum-order observer improves the performance of the active control, the parameters characterizing the observer are, respectively, determined so as to minimize the estimation error, $w - \hat{w}$ so that

$$T_1 = T_2 = 0, \quad T_3 = 0{\cdot}007, \quad T_4 = 0{\cdot}012$$

are obtained by performing the experiment. Furthermore, the parameters characterizing the proposed active control u_s are, respectively, determined so as to minimize the performance index J so that

$$u_s^* = 1{\cdot}01, \quad \varepsilon = 0{\cdot}01$$

are obtained by performing the experiment.

The time responses of the exact non-linear model and the performance index J are, respectively, evaluated by using three kinds of methods

(a) Method A: proposed active suspension based on sliding mode control.
(b) Method B: linear active suspension based on LQ control.
(c) Method C: passive suspension.

Table 1 shows the root mean square (r.m.s) values of the acceleration $\ddot{z}_1$, the velocity $\dot{z}_1$ and the displacement z_1 of the car body, the suspension displacement $(z_1 - z_2)$, the tire deflection $(z_2 - w)$ and the active control u subject to excitation from the sinusoidal road profile, and also the performance index J evaluated from these values. In order to evaluate the ride comfort due to the active control, the time response of the acceleration $\ddot{z}_1$ of the car body and its spectral density function Φ are, respectively, shown in Figures 2 and 3. Figure 4 shows the estimation of the sinusoidal road profile by using the proposed minimum order observer. It is seen from Table 1 that Method A decreases the r.m.s. values of the variables and the performance index J more than the other two methods, and it is seen from Figures 2 and 3 that the former decreases the amplitude of the time response and the peak in the neighborhood of 3 Hz more than the latter. Figure 4 shows the improved performance in the estimation of w as the amplitude and frequency of the exact and

TABLE 1

Root mean square values of the time responses of the model, and the performance index (sinusoidal road)

	Method A	Method B	Method C	Unit
$\ddot{z}_1$	$4{\cdot}76\times10^{-1}$	$5{\cdot}21\times10^{-1}$	$1{\cdot}74$	m/s^2
$\dot{z}_1$	$1{\cdot}35\times10^{-2}$	$1{\cdot}56\times10^{-2}$	$7{\cdot}87\times10^{-2}$	m/s
z_1	$8{\cdot}78\times10^{-4}$	$1{\cdot}23\times10^{-3}$	$4{\cdot}39\times10^{-3}$	m
$z_1 - z_2$	$9{\cdot}61\times10^{-4}$	$1{\cdot}16\times10^{-3}$	$1{\cdot}89\times10^{-2}$	m
$z_2 - w$	$2{\cdot}77\times10^{-4}$	$2{\cdot}85\times10^{-4}$	$3{\cdot}96\times10^{-4}$	m
u	$4{\cdot}31\times10$	$3{\cdot}21\times10$	—	N
J	$3{\cdot}47\times10^{-4}$	$3{\cdot}70\times10^{-4}$	$6{\cdot}19\times10^{-3}$	—

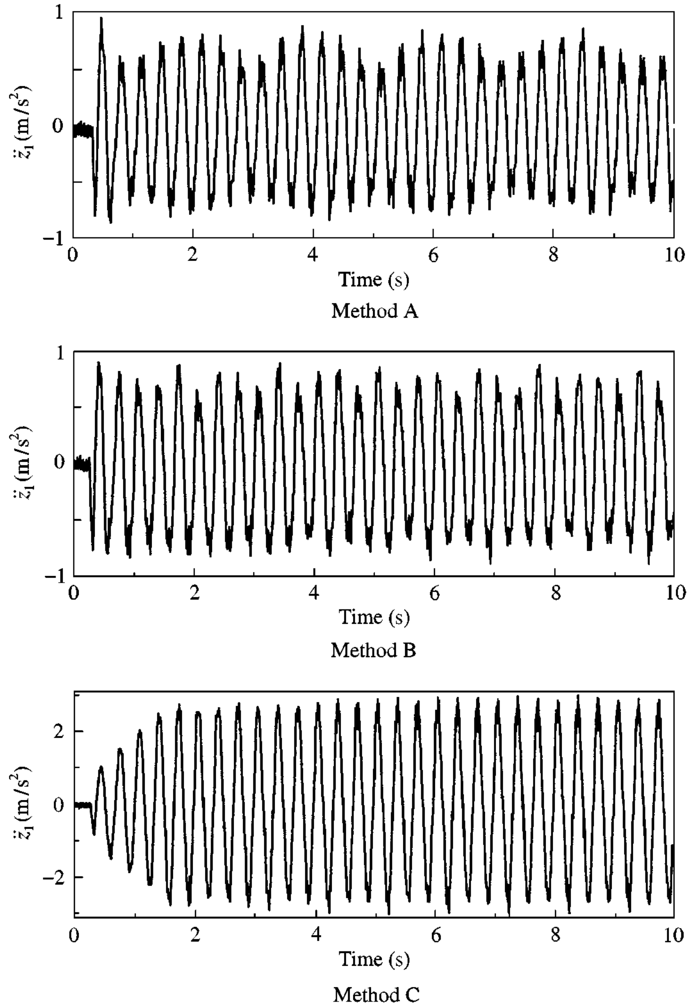

Figure 2. Time response $\ddot{z}_1$ (sinusoidal road).

TABLE 2

Root mean square values of the time responses of the model, and the performance index (random road)

	Method A	Method B	Method C	Unit
$\ddot{z}_1$	$7{\cdot}32 \times 10^{-1}$	$7{\cdot}55 \times 10^{-1}$	$1{\cdot}32$	m/s^2
$\dot{z}_1$	$1{\cdot}89 \times 10^{-2}$	$1{\cdot}93 \times 10^{-2}$	$5{\cdot}02 \times 10^{-2}$	m/s
z_1	$1{\cdot}59 \times 10^{-3}$	$1{\cdot}67 \times 10^{-3}$	$2{\cdot}80 \times 10^{-3}$	m
$z_1 - z_2$	$1{\cdot}66 \times 10^{-3}$	$1{\cdot}72 \times 10^{-3}$	$1{\cdot}82 \times 10^{-3}$	m
$z_2 - w$	$4{\cdot}64 \times 10^{-4}$	$4{\cdot}72 \times 10^{-4}$	$8{\cdot}00 \times 10^{-4}$	m
u	$4{\cdot}27 \times 10$	$4{\cdot}32 \times 10$	—	N
J	$5{\cdot}43 \times 10^{-4}$	$5{\cdot}60 \times 10^{-4}$	$2{\cdot}52 \times 10^{-3}$	—

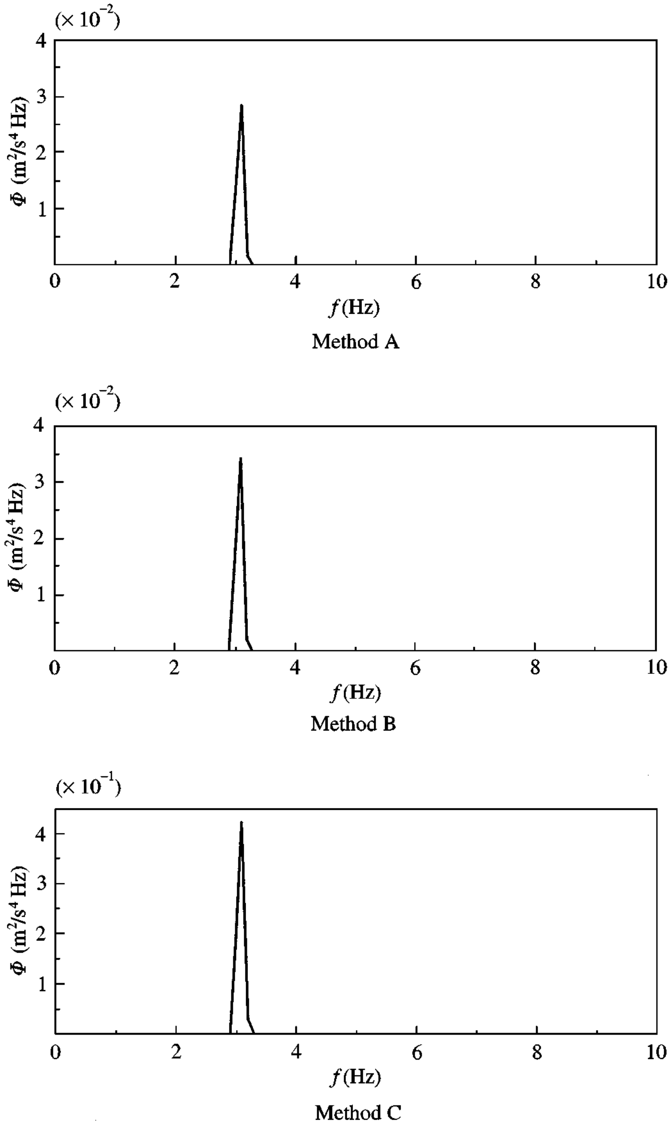

Figure 3. Spectral density function for $\ddot{z}_1$ (sinusoidal road).

estimated values are approximately identical, but the phase shift between the exact and the estimated values is slightly seen.

Table 2 shows the r.m.s. values of the identical variables as denoted in Table 1 subject to excitation from the random road profile, and the performance index J evaluated by these values. Figures 5 and 6, respectively, show the time response of the acceleration $\ddot{z}_1$ of the car body, and its spectral density function Φ, and Figure 7 shows the estimation of the random road profile by using the proposed minimum order observer. It is seen from Table 2 that Method A improves the r.m.s. values of the variables and the performance index J more than the other two methods. It is seen from Figures 5 and 6 that the former decreases the amplitude of the time response and the peaks between 2 and 6 Hz more than the latter. It is particularly noted that the active control u obtained by Method A is smaller than that

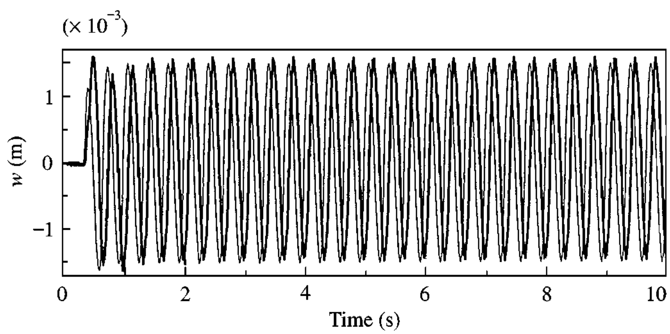

Figure 4. Estimation of the sinusoidal road w. ——, Exact; ——, Estimated.

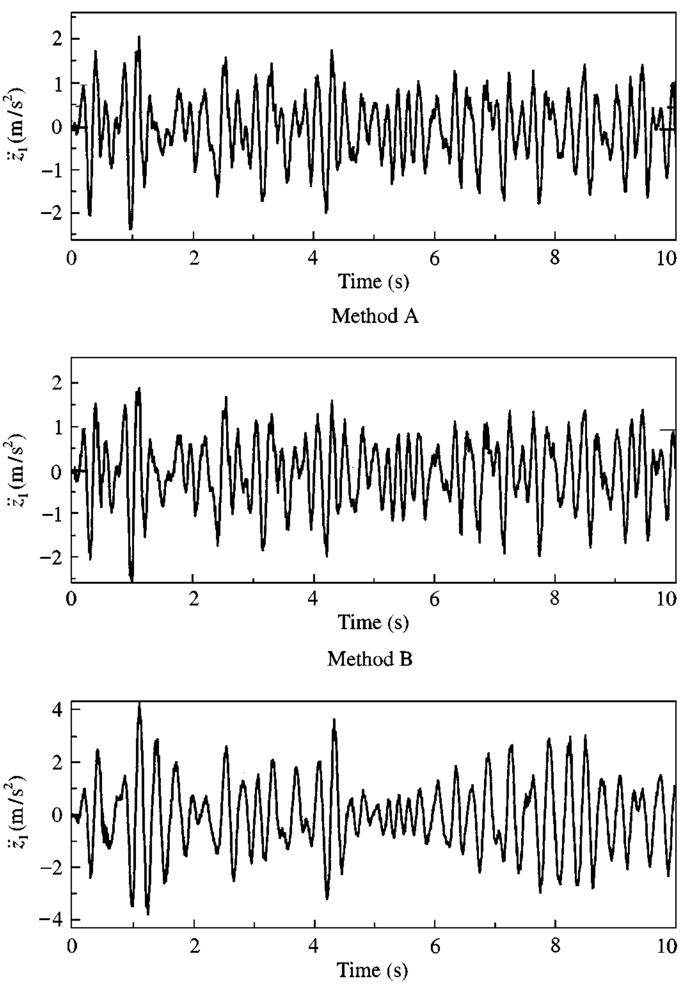

Figure 5. Time response $\ddot{z}_1$ (random road).

obtained by Method B, compared with the case of the sinusoidal road. Figure 7 shows the slightly improved performance in the estimation of w, but the estimation error becomes larger as the amplitude of the random road profile is larger.

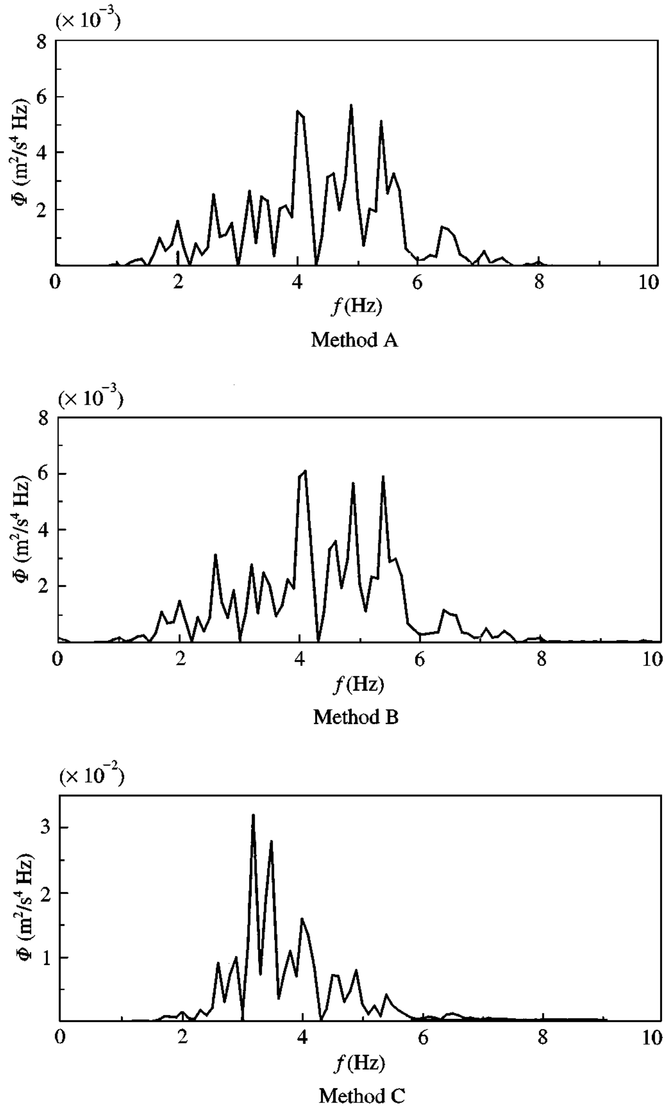

Figure 6. Spectral density function for $\ddot{z}_1$ (random road).

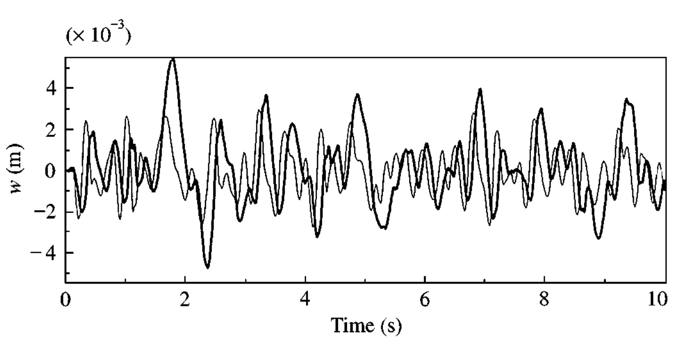

Figure 7. Estimation of the random road w. ——, Exact; ——, Estimated.

Comparing the performance of Methods A and B, the former is better than the latter in the r.m.s. values of the time responses subject to two kinds of road profiles. It means that the former is constructed by the exact non-linear system and the trajectory of the time response is restricted on the sliding surface for a large value of $|u_E|$, while the latter is only constructed by a linear system transformed from the exact non-linear system. Furthermore, comparing Method A with the conventional sliding mode control (the equivalent control adds to the non-linear switching control [8]), the experiment indicates that the former is much better than the latter in the suspension performance of the car body.

7. CONCLUSION

The paper proposed an active suspension system for a quarter car model using the concept of sliding mode control. The proposed active suspension system was constructed by the switching function whose amplitude is denoted by the absolute value of the equivalent control. The sliding surface was derived by LQ control theory where a linear system was transformed from the original nonlinear system. Two kinds of road profiles were presented in the experiment, and the road profile was estimated by using the minimum order observer. The experimental result showed that the proposed active suspension system improved the vibration isolation of the car body more than the linear active suspension system based on LQ control, and the passive-suspension system.

REFERENCES

1. A. Hác 1986 *ASME Journal of Dynamic systems, Measurement, and Control* **108**, 106–110. Stochastic optimal control of vehicle with elastic body and active suspension.
2. D. Hrovat 1993 *ASME Journal of Dynamic systems, Measurement, and Control* **115**, 328–342. Applications of optimal control to advanced automotive suspension design.
3. T. Yoshimura and M. Sugimoto 1990 *Journal of Sound and Vibration* **138**, 433–445. An active suspension for a vehicle travelling on flexible beams with an irregular surface.
4. M. Athans and P. L. Falb 1966 *Optimal Control*. New York: McGraw-Hill.
5. T. Yoshimura, Y. Isari, Q. Li and J. Hino 1997 *Control Engineering Practice* **5**, 175–184. Active suspension of motor coaches using skyhook damper and fuzzy logic controls.
6. A. Moran and M. Nagai 1994 *JSME International Journal* **37**, 707–718. Optimal active control of nonlinear vehicle suspensions using neural networks.
7. M. Kurimoto and T. Yoshimura 1998 *International Journal of Vehicle Design* **19**, 402–414. Active suspension of passenger cars using sliding mode controllers (based on reduced models).
8. K. Nonami and H. Den 1994 *Sliding Mode Control*. Tokyo: Corona (in Japanese).
9. Y. Ohtani and T. Yoshimura 1996 *International Journal of Systems Science* **27**, 179–186. Fuzzy control of a manipulator using the concept of sliding mode.
10. V. I. Utkin 1977 *IEEE Transactions of Automatic Control* **AC-12**, 212–222. Variable structure systems with sliding mode.
11. C. Edwards and S. K. Spurgeon 1998 *Sliding Mode Control: Theory and Application*. London: Taylor & Francis.
12. Z. Iwai, A. Inoue and S. Kawaji 1998 *Observer*. Tokyo: Corona. (in Japanese).