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Observer-based event-triggered control for linear MASs under a directed graph and DoS attacks

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ABSTRACT
In this paper, we investigate the observer-based event-triggered consensus problem for linear multi-agent systems (MASs) under a directed graph and denial-of-service (DoS) attacks. A type of DoS attacks launched by malicious attackers at irregular intervals is considered, which can cause communication channel disruption. A novel event-triggered secure control scheme based on a closed-loop observer is proposed to determine the scheduling of the controller update, and a separation method with less conservativeness is employed to design the controller and observer gains. Then, the frequency and duration of DoS attacks that can be tolerated are analysed for the observer-based secure consensus problem. In addition, a strictly positive minimal event-triggered time interval for each agent is designed with the help of the proposed event-triggered condition to eliminate the Zeno behaviour. Finally, a numerical simulation is given to verify the theoretical analysis.

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Directed graph; DoS attacks; event-triggered mechanism; consensus control; multi-agent systems

1. Introduction

In recent years, with the development of Unmanned Aerial Vehicle (UAV) control, underwater cooperative operation, robot formation control and other cluster control fields, the cooperative control of MASs has gradually become a research hotspot. Vicsek et al. (2006) consider the scene of multiple particles moving in the plane and use the nearest neighbour rule for local coordination to achieve a uniform overall motion. Starting from the model in Vicsek et al. (2006), the consistency of discrete-time systems with time-varying topology by using the nonnegative matrix, the switched systems and the stability theory is discussed in Jadbabaie et al. (2003). The work in Olfati-Saber and Murray (2004) analyses the convergence of directed fixed topology, directed switching topology and undirected network with time delay, and obtains a necessary and sufficient condition that the agent converges to the average consistency. Based on the above work, a lot of meaningful work have been produced (Z. Li et al., 2010, 2013; Liang et al., 2020; Ma et al., 2021b).

The disadvantage of the above work is that it needs frequent local information exchange between neighbouring agents. As we all know, unnecessary communication will cause a lot of network resource waste, and continuous communication will also cause network resource competition among agents. In order to eliminate the requirement of continuous communication, there have been many researches on the event-triggered mechanism (He et al., 2021; Liu & Yu, 2017; Ma et al., 2021a; L. Wang & Dong, 2020; Y. W. Wang et al., 2020, 2018). In Guo et al. (2014), the consistency issues of the sampling period were studied. The cooperative control of a heterogeneous MAS is investigated in Hu and Liu (2016). Particularly, in Fan et al. (2015), a self-triggered consensus algorithm for MASs is proposed, and the Zeno behaviour is excluded by specifying a strictly positive event-triggered time interval for each agent system. However, the topology considered in the above work is with an undirected graph. It should be pointed out that undirected graphs are a special case of directed graphs, so the above results cannot be applied to directed graph networks with the asymmetric Laplacian matrix. For the directed network topology, three design methods for the cooperative control problem of MASs are discussed in H. Zhang et al. (2012), namely the Lyapunov design method, the neural adaptive design method and the optimal design method based on the linear quadratic regulator (LQR). In Yu et al. (2010), a new concept of generalised algebraic connectivity is proposed for strongly connected networks, and several sufficient conditions for MASs with nonlinear dynamics are given to reach the second-order consistency. Based on Yu et al. (2010), distributed

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event-triggered control strategies of state-dependent thresholds are proposed to solve the consensus control and the finite-time consensus control problems of MASs in Liu et al. (2017) and Du et al. (2018), respectively. Focused on the event-triggered consensus control problem, a self-triggered consensus controller is designed in D. Yang et al. (2015), in which each agent only needs to continuously monitor its own state to determine when to trigger an event and broadcast its state to its neighbours. An observer-based output feedback event-triggered control scheme is investigated in Jian et al. (2019), and the consensus of the controlled MASs is realised asymptotically.

It is worth noting that the above work is based on the assumption of perfect communication networks. In the actual communication networks, there are some unavoidable factors that affect the information transmission, such as packet losses, network delay and malicious network attacks. The network fluctuations caused by these factors will seriously affect the performance of the MASs (Huang & Pan, 2017; Ren et al., 2019; C. Wei et al., 2018). In particular, malicious network attacks can disrupt communication between networks or even indirectly have a disastrous impact on the physical systems (Crdenas et al., 2011, 2009). Malicious network attacks can be roughly divided into two categories. One is deceptive attacks that aim at tampering with data and injecting error information into the network. The results of consensus problems of MASs under some deceptive attacks have been introduced in He et al. (2018) and X. M. Li et al. (2020). The other is DoS attacks that send a large amount of useless data to the network channel to prevent data transmission (Agarwal et al., 2017; Y. R. Deng et al., 2021; Liu et al., 2020; Teixeira et al., 2012). In C. Deng and Wen (2020), the distributed resilient observer-based fault-tolerant control problem is investigated for heterogeneous linear MASs with actuator faults and DoS attacks. A class of state feedback controllers for network control systems under DoS attack are studied in X. M. Zhang et al. (2020). The work in Ge et al. (2020) investigates the distributed event-triggered estimation of a dynamic system running on the sensor network with limited resources. The consensus problem of MASs under DoS attacks are considered in C. Deng et al. (2020), Y. Yang et al. (2020), Feng and Hu (2019), and Amini et al. (2020). The event-triggered output consensus problem for heterogeneous MASs with nonuniform communication delays is studied in C. Deng et al. (2020). In Y. Yang et al. (2020), the consensus problem of the MASs under DoS attacks is studied by using two-terminal event-triggered mechanisms to schedule information transmission for each follower: one on the measurement channel and the other on the control channel. The event-triggered consensus control problems of MASs with leadless and leader-follower under DoS attacks are investigated in Feng and Hu (2019), respectively. An optimised consistent elastic framework based on a dynamic event-triggered mechanism is proposed in Amini et al. (2020). However, the results obtained in Feng and Hu (2019) and Amini et al. (2020) are based on the state-feedback, and only applicable to MASs connected through an undirected graph network.

In order to overcome the shortcomings mentioned above, an observer-based event-triggered consensus method is proposed to solve the leaderless consensus problem of linear MASs with DoS attacks under a directed graph. The main contributions of this paper are as follows:

- This paper considers the event-triggered consensus control problem under the directed graph for MASs in the presence of DoS attacks in the cyber layer and the absence of the state measurement in the physical layer. To solve the problem, a novel distributed observer-based resilient event-triggered control strategy is provided.
- Unlike Liu et al. (2017), Feng and Hu (2019) and Amini et al. (2020), a novel closed-loop observer-based event-triggered mechanism is designed to reduce the cost of communication and the impact of DoS attacks on system performance. The designed event-triggered mechanism introduces a reasonable positive inter-event time to eliminate the Zeno behaviour. In addition, the controller gain and observer gain are designed by using the separation method, which reduces the conservativeness compared with H. Zhang et al. (2014) and Ruan et al. (2020).

The remainder of this paper is organised as follows. Section 2 introduces some preliminaries and problem formulation. Section 3 proposes the main result. Then, a simulation example is given to illustrate the theoretical result in Section 4. Finally, Section 5 gives conclusions of the paper.

Notation: The notations used in this paper are defined as follows: I_N represents an N-dimensional identity matrix and 0_N represents an N-dimensional zero matrix. $\mathbf{0}_N$ and $\mathbf{1}_N$ denote the $N \times 1$ column vector with all zero elements and all one elements, respectively. $\|\cdot\|$ represents the Euclidean vector norm. $X > 0$ indicates X is a positive definite matrix. $\otimes$ stands the Kronecker product. $\mathbb{R}^n$ denotes n-dimensional real number space. $\mathbb{R}^{n\times n}$ denotes the sets of all $n \times n$ real matrices. $M \setminus N$ donates the set belongs to the set M but not the set N. diag$\{\cdot\}$ stands for the diagonal matrix. $\mathbb{R}_+$ represents the positive real number. For vectors $x_i \in \mathbb{R}^n$, the vector $[x_1^T, \ldots, x_N^T]^T$ is denoted by $col\{x_1, \ldots, x_N\}$.

2. Problem formulation

2.1. Preliminaries

First, some knowledge of graph theory is demonstrated. The topology graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ represents the interaction among the MASs, where $\mathcal{V} = \{v_1, \ldots, v_N\}$

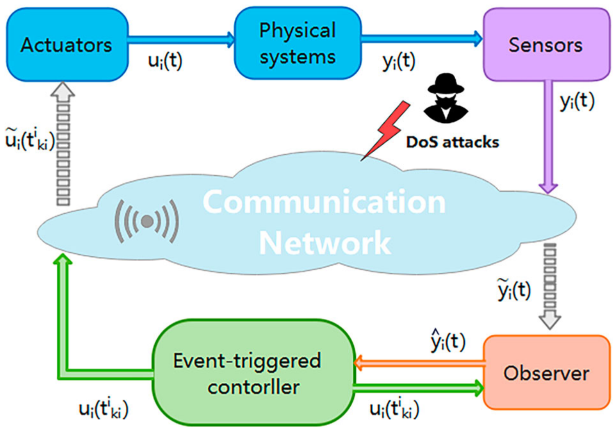

Figure 1. Framework of MASs under DoS attacks.

is the set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the set of edges. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is represented with $a_{ii} = 0$ and $a_{ij} = 1$ if $(v_i, v_j) \in \mathcal{E}$, otherwise $a_{ij} = 0$. A directed edge $\mathcal{E}$ in graph $\mathcal{G}$ is denoted by the ordered pair of nodes (v_i, v_j), where v_i and v_j are called the initial and terminal nodes, respectively, which means that node v_j can receive information from node v_i. The neighbourhood of the agent i is defined as $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}\}$. The Laplacian matrix $L = [L_{ij}] \in \mathbb{R}^{N\times N}$ is defined with $L_{ii} = \sum_{j=1,j\neq i}^{N} a_{ij}$ and $L_{ij} = -a_{ij} (i \neq j)$.

The following lemmas and assumptions will be used in this paper.

Assumption 2.1: The considered directed network topology $\mathcal{G}$ is strongly connected.

Lemma 2.1 (R. Wei & Beard, 2005): *If Assumption 2.1 is satisfied, then we have (i) $L\mathbf{1}_N = \mathbf{0}_N$; (ii) there is a vector $\xi = [\xi_1, \xi_2, \ldots, \xi_N]^T$ with $\xi_i > 0$ $(i = 1, 2, \ldots, N)$ and $\xi\mathbf{1}_N = 1$ such that $\xi^T L = \mathbf{0}_N^T$.*

Definition 2.2 (Yu et al., 2010): For a strongly connected network with Laplacian matrix L, the algebraic connectivity is defined by

$$\mathrm{a}(L) = \min_{x^T\xi=0, x\neq 0} \frac{x^T\hat{L}x}{x^T\Xi x}, \tag{1}$$

where $\hat{L} = (L^T\Xi + \Xi L)/2$ with $\Xi = \mathrm{diag}\{\xi_1, \xi_2, \ldots, \xi_N\}$.

2.2. System model

Firstly, Figure 1 is presented to depict the system structure of this paper, which consists of sensors, actuators, controllers, observers and event-triggered mechanisms.In Figure 1, $\tilde{u}_i(t^i_{k_i})$ and $\tilde{y}_i(t)$ represent control input signals and measurement output signals under DoS attacks, respectively.

Consider a multiagent network consisting of N agents with identical general linear dynamics. The dynamics of the ith agent are described by

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t), & t \in \mathbb{R}_+, \\ y_i(t) = Cx_i(t), \end{cases} \tag{2}$$

where $x_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^m$ are the state and control input vectors, respectively. $y_i(t) \in \mathbb{R}^d$ is the measurement output of the ith agent. A, B and C are given constant matrices with rational dimensions. It is assumed that the matrix pair (A, B) is stabilisable and the matrix pair (A, C) is observable.

Due to the fact that not all system states are measurable in practical systems, so an observer-based event-triggered control strategy is proposed in this part.

Firstly, considering the following observer:

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + Bu_i(t) + G(y_i(t) - \hat{y}_i(t)), \\ \hat{y}_i(t) = C\hat{x}_i(t), \end{cases} \tag{3}$$

where $\hat{x}_i(t) \in \mathbb{R}^n$ is the observer state and $G \in \mathbb{R}^{n\times d}$ is the observer gain to be determined.

To save network resources, an event-triggered mechanism is introduced to update control signals at event-triggered instants. Suppose that the event-triggered sequence for agent i is determined as $\{t^i_{k_i}\}$ $(k_i = 0, 1, \ldots)$. Then, the following event-triggered condition is proposed:

$$g_i(t) = \|\tilde{m}_i(t)\| - \beta_i\|y_i(t) - \hat{y}_i(t)\|, \tag{4}$$

where β_i is a positive scalar to be determined. $\tilde{m}_i(t) = \sum_{j=1}^{N} a_{ij}(m_i(t) - m_j(t))$, where $m_i(t) = \hat{x}_i(t) - \hat{x}_i(t^i_{k_i})$ is the measurement error with $\hat{x}_i(t^i_{k_i})$ being the observer state at the latest event-triggered time $t^i_{k_i}$ of agent i.

The control signal will be updated when $g_i(t) > 0$. Inspired by Fan et al. (2015), the following hybrid event-triggered mechanism is introduced

$$t^i_{k+1} = \begin{cases} t^i_{k_i} + \vartheta_i, & \text{if } t^i_{k_i} \in \Xi_a(0, t), \\ t^i_{k_i} + \triangle^i_{k_i}, & \text{otherwise}, \end{cases}$$

$$\triangle^i_{k_i} = \max\{\tau^i_{k_i}, b_i\}, \tag{5}$$

where ϑ_i is a positive constant and $\Xi_a(0, t)$ denotes the set of time intervals when the communication is denied under DoS attacks, which will be discussed in Section 2.3. $t^i_{k_i}$ is the latest successful event-triggered time of agent i, $i = 1, 2 \ldots, N$, $k_i = 0, 1, \ldots$. $\triangle^i_{k_i}$ is the event-triggered interval, b_i is a positive scalar introduced to facilitate eliminating Zeno behaviour, which will be determined in Section 3, and

$$\tau^i_{k_i} = \inf_{t > t^i_{k_i}} \{t - t^i_{k_i} | g_i(t) > 0\}.$$

Then, in order to obtain consensus, the following observer-based event-triggered controller is designed

$$u_i(t) = cK \sum_{j\in\mathcal{N}_i(\mathcal{G})} a_{ij}(\hat{x}_i(t^i_{k_i}) - \hat{x}_j(t^j_{k_j})), \tag{6}$$

where $c > 0$ is the coupling gain to be designed, and $K \in \mathbb{R}^{m\times n}$ is the controller gain to be designed.

Denote the stack vectors as

$$\begin{aligned}
x(t) &= col\{x_1^T(t), \dots, x_N^T(t)\},\\
\hat{x}(t) &= col\{\hat{x}_1^T(t), \dots, \hat{x}_N^T(t)\},\\
e(t) &= col\{e_1^T(t), \dots, e_N^T(t)\},\\
m(t) &= col\{m_1^T(t), \dots, m_N^T(t)\},\\
\tilde{m}(t) &= col\{\tilde{m}_1^T(t), \dots, \tilde{m}_N^T(t)\}.
\end{aligned}$$

According to (2), (3) and (6), the closed-loop system can be written as

$$\begin{aligned}
\dot{x}(t) &= (I_N \otimes A + cL \otimes BK)x(t)\\
&\quad - (cL \otimes BK)(e(t) + m(t)),\\
\dot{\hat{x}}(t) &= (I_N \otimes A)\hat{x}(t) + (cL \otimes BK)(x(t) - e(t) - m(t))\\
&\quad + (I_N \otimes GC)e(t),
\end{aligned} \tag{7}$$

where $e(t) = x(t) - \hat{x}(t)$ denotes the observer error. Based on (7), we have

$$\dot{e}(t) = (I_N \otimes (A - GC))e(t). \tag{8}$$

Motivated by Ren (2008), define the disagreement vector as $\delta(t) = x(t) - (\mathbf{1}_N\xi^T \otimes I_n)x(t) = [(I_N - \mathbf{1}_N\xi^T) \otimes I_n]x(t) = (\mathcal{M} \otimes I_n)x(t)$. By using Lemma 2.1, we have $\mathcal{M}L = L = L\mathcal{M}$. Thus, we obtain

$$\begin{aligned}
\dot{\delta}(t) &= (I_N \otimes A + cL \otimes BK)\delta(t)\\
&\quad - (cL \otimes BK)(e(t) + m(t)).
\end{aligned} \tag{9}$$

Define $z(t) = [\delta^T(t), e^T(t)]^T$. The following compact form can be obtained:

$$\dot{z}(t) = Wz(t) + M\tilde{m}(t), \tag{10}$$

where $W = \begin{bmatrix} I_N\otimes A + cL\otimes BK & -cL\otimes BK \\ 0 & I_N\otimes(A-GC)\end{bmatrix}$ and $M = \begin{bmatrix} -cI_N\otimes BK \\ 0\end{bmatrix}$.

Now, the leaderless consensus problem of MASs is transformed into the stability problem of $z(t)$ under the event-triggered control strategy.

Remark 2.1: β_i is a parameter related to the communication frequency and convergence rate. For example, the smaller β_i is selected, the faster the convergence and the more frequent communication are required. ϑ_i in (5) is a positive constant related to the exact information of DoS attacks. In fact, both the controller and the observer are invalid under DoS attacks. To reduce the number of unnecessary triggers during the DoS attacks, a large enough positive constant ϑ_i is firstly selected.

Remark 2.2: Unlike the assumption that the real-time state information of neighbours is available by each agent in the existing results (Amini et al., 2020; Feng & Hu, 2019; Liu et al., 2017), the event-triggered condition proposed in this paper does not require the system state, and each agent sends control signals by monitoring the estimated state of its neighbours.

2.3. DoS attacks model

It is assumed that the DoS attacks sequence is $\{\bar{t}_l\}_{l\in\mathbb{N}}$, where $\bar{t}_l$ is the lth DoS attack start time. The lth DoS attacks interval is defined as $[\bar{t}_l, \bar{t}_l + \bar{\Delta}_l)$ with $\bar{\Delta}_l$ being the duration of DoS attacks, and $\bar{t}_{l+1} > \bar{t}_l + \bar{\Delta}_l$. Then, we can obtain that the total time interval of invalid communication affected by DoS attacks is determined as $\Xi_a(t_0, t) = \cup_{l\in\mathbb{N}}[\bar{t}_l, \bar{t}_l + \bar{\Delta}_l) \cap [t_0, t]$. The total time interval without DoS attacks during $[t_0, t]$ is determined as $\Xi_s(t_0, t) = [t_0, t] \setminus \Xi_a(t_0, t)$.

It should be noted that Δ_* is defined as the upper bound of two consecutive event-triggered time intervals (Feng & Hu, 2019), i.e. $\Delta_* = \sup\{t^i_{k_i+1} - t^i_{k_i}\}$. That is, there has no control signal update in Δ_*, so, the 'actual effective' DoS attacks time interval is $[\bar{t}_l, \bar{t}_l + \bar{\Delta}_l + \Delta_*)$. Further, the total time interval without control signal transmission is obtained as

$$\bar{\Xi}_a(t_0, t) = \cup_{l\in\mathbb{N}}[\bar{t}_l, \bar{t}_l + \bar{\Delta}_l + \Delta_*) \cap [t_0, t],\ l \in \mathbb{N}. \tag{11}$$

Accordingly, $\bar{\Xi}_s(t_0, t) = [t_0, t] \setminus \bar{\Xi}_a(t_0, t)$ represents the total valid communication time interval.

Definition 2.3 (Feng & Hu, 2014, Attack Frequency): Define $n_a(t_0, t)$ as the amount of DoS attacks occurring in $[t_0, t)$, then, the frequency of DoS attacks over $[t_0, t)$ is defined as follows:

$$F_a(t_0, t) = \frac{n_a(t_0, t)}{t - t_0}. \tag{12}$$

Definition 2.4 (Feng & Hu, 2014, Attack Duration): Define $|\Xi_a(t_0, t)|$ as the total time interval in the presence of DoS attacks in $[t_0, t)$, then, the duration of DoS attacks satisfies

$$|\Xi_a(t_0, t)| \le \Xi_0 + \frac{t - t_0}{\tau_a}, \tag{13}$$

where $\tau_a > 1$ and Ξ_0 are scalars to be determined.

Remark 2.3: Definitions 2.3 and 2.4 are used in Y. Yang et al. (2020) and Feng and Hu (2019) to analyse the DoS attacks model. The frequency of DoS attacks is specified in Definition 2.3, and the upper bound will be analysed below. In Definition 2.4, τ_a represents the strength of DoS attacks and the role of Ξ_0 is to consider the attack at the start time.

2.4. Control objective

The following problem is given which will be used in Section 3.

Observer – Based Secure Consensus Problem (OBSCP) : For MASs composed of dynamic (2) with DoS attacks under the directed graph, the goal of this paper is to design an event-triggered controller (6) based on the observer (3) to guarantee that for $\forall i, j \in \mathcal{V}$, there exist a scalar $\varpi > 0$ and a decay rate $\iota > 0$ such that

$$\|x_i(t) - x_j(t)\|^2 \le \varpi e^{-\iota(t-t_0)}, \quad \forall t > t_0. \tag{14}$$

3. Main results

It is noted that the actuator and observer will not be able to receive the control input signals from the controller and the measurement output signals from the sensor when the system suffers from DoS attacks, that is, the system will be transformed into an open-loop system. Then, combined with the DoS attack model described in Section 2.3, (10) can be transformed into the following form:

$$\begin{cases} \dot{z}(t) = Wz(t) + Mm(t), & t \in \bar{\Xi}_s(t_0, \infty), \\ \dot{z}(t) = \bar{W}z(t), & t \in \bar{\Xi}_a(t_0, \infty), \end{cases} \tag{15}$$

where W and M are defined in (10) and $\bar{W} = \begin{bmatrix} I_N \otimes A & 0 \\ 0 & I_N \otimes A \end{bmatrix}$.

Next, the main results are presented as follows.

Theorem 3.1: *For given controller gain K, observer gain G and scalars $\kappa_1 > 0$, $\kappa_2 > 0$, $\beta_i < \frac{\gamma_1}{\|C\|}$, $\eta_1^* \in (0, \kappa_1)$, with Assumption 2.1, the considered OBSCP can be solved, if there exist positive definite matrices P, Q and positive scalar $\gamma = \gamma_1 + \gamma_2$ with $\gamma_1 \ge \max\{\beta_i\}\|C\|$, $\gamma_2 > 0$, $b_i \le \mathcal{B}$ such that the following inequalities are satisfied:*

$$\Lambda + \kappa_1 \mathcal{P} < 0, \tag{16}$$

$$\mathcal{P}\bar{W} + \bar{W}^T\mathcal{P} - \kappa_2\mathcal{P} < 0, \tag{17}$$

$$F_a(t_0, t) - \frac{\eta_1^*}{(\kappa_1 + \kappa_2)\Delta_*} \le 0, \tag{18}$$

$$-\tau_a + \frac{\kappa_1 + \kappa_2}{\kappa_1 - \eta_1^*} < 0, \tag{19}$$

where

$$\Lambda = \begin{bmatrix} \Theta + \gamma^2(I_N \otimes I_n) & 0 \\ 0 & \Pi + \gamma^2(I_N \otimes I_n) \end{bmatrix},$$

$$\mathcal{B} = \frac{1}{\sigma}\ln\left(1 + \frac{d}{2\|L\|}\right), \quad \mathcal{P} = \mathrm{diag}\{\Xi \otimes P, I_N \otimes Q\} \tag{20}$$

with

$$\Theta = \Xi \otimes (PA + A^TP - ca(L)PBB^TP) + (\lambda_{\max}(LL^T) + 1) \times \left(\frac{1}{2}c\Xi \otimes PBB^TP\right)\left(\frac{1}{2}c\Xi \otimes PBB^TP\right)^T,$$

$$\Pi = I_N \otimes (QA + A^TQ - QGC - (QGC)^T + I_n),$$

$$d = \frac{\gamma_2}{N}, \quad \sigma = \|W\| + N\|M\|d. \tag{21}$$

Proof: The proof will be done from the following two aspects: the stability analysis of the closed-loop system and the exclusion of Zeno behaviour.

(1) *Stability analysis*

In this part, we divide the total time interval $[t_0, t)$ into two parts: the time interval $\bar{\Xi}_s(t_0, t)$ where the event-triggered function (4) holds and the time interval $\bar{\Xi}_a(t_0, t)$ where (4) does not hold.

Consider the Lyapunov function

$$V(t) = z^T(t)\mathcal{P}z(t). \tag{22}$$

For $t \in \bar{\Xi}_s(t_0, \infty)$, the derivative of $V(t)$ is calculated according to (15) as follows:

$$\begin{aligned} \dot{V}(t) &= 2z^T(t)\mathcal{P}\dot{z}(t) \\ &= 2z^T(t)\mathcal{P}Wz(t) + 2z^T(t)\mathcal{P}M\tilde{m}(t). \end{aligned} \tag{23}$$

Using $K = -\frac{1}{2}B^TP$, we can rewrite the first part of (23) as

$$\begin{aligned} 2z^T(t)\mathcal{P}Wz(t) = {} & \delta^T(t)(\Xi \otimes (PA + A^TP) \\ & - \frac{1}{2}c(\Xi L + L^T\Xi) \otimes PBB^TP)\delta(t) \\ & + 2\delta^T(t)\left(c\Xi L \otimes \frac{1}{2}PBB^TP\right)e(t) \\ & + 2e^T(t)(I_N \otimes (QA - QGC))e(t). \end{aligned} \tag{24}$$

By Definition 2.2 and using Young's inequality $a^Tb + b^Ta \le a^Ta + b^Tb$ with $a = (c\Xi L \otimes \frac{1}{2}PBB^TP)\delta(t)$ and $b = e(t)$, one gets

$$\begin{aligned} 2z^T(t)\mathcal{P}Wz(t) \le {} & \delta^T(t)(\Xi \otimes (PA + A^TP \\ & - \mathrm{ca(L)}PBB^TP))\delta(t) \\ & + \delta^T(t)\left(c\Xi LL^T\Xi c \otimes \frac{1}{2}PBB^TP \right.\\ & \left.\times \left(\frac{1}{2}PBB^TP\right)^T\right)\delta(t) \\ & + e^T(t)(I_N \otimes (QA + A^TQ \\ & - QGC - (QGC)^T + I_n))e(t) \\ \le {} & e^T(t)\Pi e(t) + \delta^T(t) \\ & \times \left(\Xi \otimes (PA{+}A^TP{-}\mathrm{ca(L)}PBB^TP)\right. \\ & + \lambda_{\max}(LL^T)\left(c\Xi \otimes \frac{1}{2}PBB^TP\right) \\ & \left.\times \left(c\Xi \otimes \frac{1}{2}PBB^TP\right)^T\right)\delta(t), \end{aligned} \tag{25}$$

where $\Pi = I_N \otimes (QA + A^TQ - QGC - (QGC)^T + I_n)$.

Similarity, taking $z(t)$, $\mathcal{P}$ and M into the second part of (23), by using Young's inequality $a^Tb + b^Ta \le a^Ta + b^Tb$, we have

$$\begin{aligned}2z^T(t)\mathcal{P}M\tilde{m}(t) &= -2\delta^T(t)\left(-\frac{1}{2}c\Xi \otimes PBB^TP\right)\tilde{m}(t)\\ &\le \delta^T(t)\left(\frac{1}{2}c\Xi \otimes PBB^TP\right)\\ &\quad\times\left(\frac{1}{2}c\Xi \otimes PBB^TP\right)^T\\ &\quad\times\delta(t) + \tilde{m}^T(t)\tilde{m}(t).\end{aligned} \tag{26}$$

Define $I_\delta = [0_N, I_N]$. If we can guarantee that

$$\|\tilde{m}_i(t)\| \le \gamma\|z_i(t)\|, \tag{27}$$

then we can obtain

$$\tilde{m}^T(t)\tilde{m}(t) \le \gamma^2 z^T(t)z(t). \tag{28}$$

It is worth pointing out that inequality (27) will be guaranteed in the later proof of estimating the Zeno behaviour. Then, one can obtain the following inequality according to (23)–(26):

$$\begin{aligned}\dot{V}(t) &\le e^T(t)\Pi e(t) + \tilde{m}^T(t)\tilde{m}(t)\\ &\quad+\delta^T(t)\left(\Xi \otimes (PA + A^TP - ca(L)PBB^TP)\right.\\ &\quad+(\lambda_{\max}(LL^T)+1)\left(\frac{1}{2}c\Xi \otimes PBB^TP\right)\\ &\quad\left.\times\left(\frac{1}{2}c\Xi \otimes PBB^TP\right)^T\right)\delta(t)\\ &= \delta^T(t)\Theta\delta(t) + e^T(t)\Pi e(t) + \tilde{m}^T(t)\tilde{m}(t).\end{aligned} \tag{29}$$

According (28), (29) can be rewritten as

$$\begin{aligned}\dot{V}(t) &\le \delta^T(t)\Theta\delta(t) + e^T(t)\Pi e(t) + \tilde{m}^T(t)\tilde{m}(t)\\ &= z^T(t)\Lambda z(t).\end{aligned} \tag{30}$$

Based on (16), the time derivative of $V(t)$ can be deduced that

$$\dot{V}(t) \le z^T(t)\Lambda z(t) \le -\kappa_1 z^T(t)\mathcal{P}z(t) = -\kappa_1 V(t). \tag{31}$$

On the other hand, for $t \in \bar{\Xi}_a(t_0,\infty)$, one gets

$$\dot{V}(t) = z^T(t)(\mathcal{P}\bar{W} + \bar{W}^T\mathcal{P})z(t). \tag{32}$$

According to (17), we have

$$\dot{V}(t) \le \kappa_2 V(t). \tag{33}$$

Let $[\bar{t}_{l-1} + \bar{\Delta}_{l-1}, \bar{t}_l) \triangleq \Phi_l$ and $[\bar{t}_l, \bar{t}_l + \bar{\Delta}_l + \Delta_*) \triangleq \Psi_l$, then, by (32), (33) and according to Khalil (2002), one can get

$$V(t) \le \begin{cases} e^{-\kappa_1(t-\bar{t}_{l-1}-\bar{\Delta})}V(\bar{t}_{l-1}+\bar{\Delta}_{l-1}), & t \in \Phi_l,\\ e^{\kappa_2(t-\bar{t}_l)}V(\bar{t}_l), & t \in \Psi_l.\end{cases} \tag{34}$$

If $t \in \Phi_l$, then, (34) can be rewritten as follows:

$$\begin{aligned}V(t) &\le e^{-\kappa_1(t-\bar{t}_{l-1}-\bar{\Delta}_{l-1})}V(\bar{t}_{l-1}+\bar{\Delta}_{l-1})\\ &\le e^{-\kappa_1(t-\bar{t}_{l-1}-\bar{\Delta}_{l-1})}V(\bar{t}_{l-1}^- + \bar{\Delta}_{l-1}^-)\\ &\le e^{-\kappa_1(t-\bar{t}_{l-1}-\bar{\Delta}_{l-1})}\\ &\quad\times[e^{\kappa_2}(t-\bar{t}_{l-2}-\bar{\Delta}_{l-2})V(\bar{t}_{l-2}+\bar{\Delta}_{l-2})]\\ &\le \dots\\ &\le e^{-\kappa_1|\bar{\Xi}_s(t_0,t)|}e^{\kappa_2|\bar{\Xi}_a(t_0,t)|}V(t_0).\end{aligned} \tag{35}$$

If $t \in \Psi_l$, then, (34) can be rewritten as follows:

$$\begin{aligned}V(t) &\le e^{\kappa_2(t-\bar{t}_l)}V(\bar{t}_l) \le e^{\kappa_2(t-\bar{t}_l)}V(\bar{t}_l^-)\\ &\le e^{\kappa_2(t-\bar{t}_l)}[e^{-\kappa_1(\bar{t}_l-\bar{t}_{l-1}-\bar{\Delta}_{l-1})}V(\bar{t}_{l-1}+\bar{\Delta}_{l-1})]\\ &\le \cdots\\ &\le e^{-\kappa_1|\bar{\Xi}_s(t_0,t)|}e^{\kappa_2|\bar{\Xi}_a(t_0,t)|}V(t_0).\end{aligned} \tag{36}$$

It is not difficult to obtain $|\bar{\Xi}_s(t_0,t)| = t - t_0 - |\bar{\Xi}_a(t_0,t)|$ and $|\bar{\Xi}_a(t_0,t)| \le |\Xi_a(t_0,t)| + (1 + n_a(t_0,t))\Delta_*$ for all $t \ge t_0$, then $|\bar{\Xi}_s(t_0,t)| \ge t - t_0 - |\Xi_a(t_0,t)| + (1 + n_a(t_0,t))\Delta_*$. According to (35) and (36), one can obtain the following inequality:

$$\begin{aligned}V(t) &\le e^{-\kappa_1|\bar{\Xi}_s(t_0,t)|}e^{\kappa_2|\bar{\Xi}_a(t_0,t)|}V(t_0)\\ &= e^{-\kappa_1(t-t_0-|\bar{\Xi}_a(t_0,t)|)}e^{\kappa_2|\bar{\Xi}_a(t_0,t)|}V(t_0)\\ &\le e^{-\kappa_1(t-t_0)+(\kappa_1+\kappa_2)|\Xi_a(t_0,t)|}\\ &\quad\times e^{(\kappa_1+\kappa_2)(1+n_a(t_0,t))\Delta_*}V(t_0)\\ &= e^{-\kappa_1(t-t_0)+(\kappa_1+\kappa_2)\frac{t-t_0}{\tau_a}}\\ &\quad\times e^{(\kappa_1+\kappa_2)(\Xi_0+(1+n_a(t_0,t))\Delta_*)}V(t_0)\\ &= e^{(\kappa_1+\kappa_2)(\Xi_0+\Delta_*)}\\ &\quad\times e^{(-\kappa_1+\frac{\kappa_1+\kappa_2}{\tau_a})(t-t_0)}\\ &\quad\times e^{n_a(t_0,t)(\kappa_1+\kappa_2)\Delta_*}V(t_0).\end{aligned} \tag{37}$$

Since $F_a(t_0,t) = \frac{n_a(t_0,t)}{(t-t_0)} \le \frac{\eta_1^*}{(\kappa_1+\kappa_2)\Delta_*}$ and $\tau_a > \frac{\kappa_1+\kappa_2}{\kappa_1-\eta_1^*}$, we can obtain

$$\begin{aligned}V(t) &\le e^{(\kappa_1+\kappa_2)(\Xi_0+\Delta_*)}e^{(-\kappa_1+\frac{\kappa_1+\kappa_2}{\tau_a})(t-t_0)}e^{\eta_1^*(t-t_0)}V(t_0)\\ &= e^{(\kappa_1+\kappa_2)(\Xi_0+\Delta_*)}e^{(-\kappa_1+\frac{\kappa_1+\kappa_2}{\tau_a}+\eta_1^*)(t-t_0)}V(t_0)\\ &\le e^{(\kappa_1+\kappa_2)(\Xi_0+\Delta_*)}e^{-\eta_1(t-t_0)}V(t_0),\end{aligned} \tag{38}$$

where $\eta_1 = \kappa_1 - \frac{\kappa_1+\kappa_2}{\tau_a} - \eta_1^* > 0$.

(2) *Eliminating the Zeno behaviour*

It is obvious that the event-triggered time interval is specified by $\tau_{k_i}^i$ or b_i according to (5). In the following, we give the lower bound of event-triggered time intervals, which can eliminate the Zeno behaviour. Let $\mathcal{W}_1(t)$ be the agent sets in which the event-triggered time interval is determined by $\tau^i_{k_i}$ and $\mathcal{W}_2(t)$ be the agent sets in which the event-triggered time interval is determined by b_i, respectively. Then, we can obtain that $\mathcal{W}_1(t) \cup \mathcal{W}_2(t) = \{0, 1, \ldots, N\}$ and $\mathcal{W}_1(t) \cap \mathcal{W}_2(t) = \emptyset$. To ensure (27), we can select $\gamma = \gamma_1 + \gamma_2$ with $\gamma_1 > 0$ and $\gamma_2 > 0$ such that

$$\sum_{i\in\mathcal{W}_1} \|\tilde{m}_i(t)\| \le \gamma_1 \sum_{i\in\mathcal{W}_1} \|z_i(t)\| \le \gamma_1 \sum_{i=1}^{N} \|z_i(t)\|, \tag{39}$$

$$\sum_{i\in\mathcal{W}_2} \|\tilde{m}_i(t)\| \le \gamma_2 \sum_{i\in\mathcal{W}_2} \|z_i(t)\| \le \gamma_2 \sum_{i=1}^{N} \|z_i(t)\|. \tag{40}$$

From the event-triggered condition $g_i(t)$ in (4), we can get $\|\tilde{m}_i(t)\| \le \beta_i \|Ce_i(t)\| \le \beta_i \|C\| \|z_i(t)\|$. So, a sufficient condition to ensure (39) is that for each agent in $\mathcal{W}_1(t)$, $\gamma_1 \ge \max\{\beta_i\}\|C\|$. Meanwhile, for agents in $\mathcal{W}_2(t)$, a sufficient condition to ensure (40) can be obtained as follows:

$$\|\tilde{m}_i(t)\| \le \frac{\gamma_2}{N} \sum_{i=1}^{N} \|z_i(t)\| = d\|z(t)\|, \tag{41}$$

where $d = \frac{\gamma_2}{N}$.

If we can obtain a lower bound time interval of event-triggered, e.g. b_i, for the evolution time of $\frac{\|\tilde{m}_i(t)\|}{\|z(t)\|}$ from 0 to d for any agent in $\mathcal{W}_2(t)$, $t^i_{k+1} = t^i_k + b_i$ can ensure (40). Then, the event-triggered time interval is verified by estimating the time derivative of $\frac{\|\tilde{m}_i(t)\|}{\|z(t)\|}$:

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\frac{\|\tilde{m}_i(t)\|}{\|z(t)\|} &= \frac{\mathrm{d}}{\mathrm{d}t}\frac{(\tilde{m}_i^T(t)\tilde{m}_i(t))^{\frac{1}{2}}}{(z^T(t)z(t))^{\frac{1}{2}}} \\
&= \frac{\tilde{m}_i^T(t)\dot{\tilde{m}}_i(t)}{\|\tilde{m}_i(t)\|\|z(t)\|} - \frac{\|\tilde{m}_i(t)\| z^T(t)\dot{z}(t)}{\|z(t)\|^3} \\
&\le \frac{\|(L\otimes I_n)(\mathcal{M}\otimes I_n)(\dot{x}(t) - \dot{e}(t))\|}{\|z(t)\|} \\
&\quad + \frac{\|\tilde{m}_i(t)\|}{\|z(t)\|}\frac{\|\dot{z}(t)\|}{\|z(t)\|} \\
&\le \frac{\|L\|(\|\dot{\delta}(t)\| + \|\dot{e}(t)\|)}{\|z(t)\|} + \frac{\|\tilde{m}_i(t)\|}{\|z(t)\|}\frac{\|\dot{z}(t)\|}{\|z(t)\|} \\
&\le \sigma\left(2\|L\| + \frac{\|\tilde{m}_i(t)\|}{\|z(t)\|}\right),
\end{aligned}$$

where $\sigma = \|W\| + Nd\|M\|$.

Notice that $\frac{\|\tilde{m}_i(t)\|}{\|z(t)\|}$ satisfies the bound $\frac{\|\tilde{m}_i(t)\|}{\|z(t)\|} < \mathcal{B}$ with $\mathcal{B} = \frac{1}{\sigma}\ln(1 + \frac{d}{2\|L\|})$ being the solution of $\dot{f}^* = \sigma(2\|L\| + f^*)$. Then, $\mathcal{B}$ is an upper bound of $\frac{\|\tilde{m}_i(t)\|}{\|z(t)\|}$ evolved from 0 to d. So, $b_i < \mathcal{B}$ guarantees (40) holding for agents in $\mathcal{W}_2(t)$.

Then controller (6) with event-triggered function (5) ensures that (38) is satisfied for agents in $\mathcal{W}_1(t) \cup \mathcal{W}_2(t)$, which means Lyapunov function (22) converges to zero. Let $\varpi_1 = e^{(\kappa_1+\kappa_2)(T_0+\Delta_*)}$. Thus, by (38), it has $\|z_i(t)\|^2 \le \varpi_1 e^{-\eta_1(t-t_0)}\|z_i(t_0)\|^2$. So, $\|\delta_i(t)\|^2 \le \|z_i(t)\|^2 \to 0$ as $t \to +\infty$, which means $x_i - x_j \to \mathbf{0}_n$ as $t \to +\infty$. Therefore, the secure average consensus is achieved exponentially for MAS (2). This accomplishes the proof. ■

Remark 3.1: In Theorem 3.1, we give the upper bound of the minimum event-triggered interval b_i under the premise of satisfying the stability condition, so as to avoid the Zeno behaviour. When MASs gradually reach consensus, the event-triggered mechanism will be executed according to b_i. The larger the selection of b_i, the less is the number of control signal updates and the slower is the consensus speed. The size of b_i can be selected according to the actual needs.

It is worth pointing out that matrix inequalities (16) and (17) are not convex and hard to deal with. To solve this problem, the convex controller and observer design conditions are given in the following theorem by using the separation method.

Theorem 3.2: *Considering MAS (2) with Assumption 2.1, for given scalars $\kappa_1 > 0, \kappa_2 > 0, \varepsilon_1 > 0, \varepsilon_2 > 0$, $\|C\|\beta_i < \gamma_1$, $\eta_1^* \in (0, \kappa_1)$, the OBSCP can be solved, if there exist matrices $X > 0$, $Y > 0$ and scalars $\gamma = \gamma_1 + \gamma_2$ with $\gamma_1 \ge \max\{\beta_i\}\|C\|$, $\gamma_2 > 0$, $b_i \le \mathcal{B}$ so that the following inequalities are satisfied for $i = 1, \ldots, N$:*

$$\begin{bmatrix} \begin{matrix}\Xi\otimes(AX + XA^T \\ +\kappa_1 X - ca(L)BB^T) \\ +I_N\otimes(\varepsilon_1 X)\end{matrix} & -\frac{1}{2}c\Xi\otimes BB^T \\ * & -\Xi\otimes(\varepsilon_2 X)\end{bmatrix} < 0, \tag{42}$$

$$\begin{bmatrix} I_N\otimes(-\varepsilon_1 I_n + \gamma^2 X) & 0 \\ * & \Xi\otimes\left(\varepsilon_2 I_n - \dfrac{1}{(\lambda_{\max}(LL^T)+1)}X\right)\end{bmatrix} < 0, \tag{43}$$

$$\Upsilon < 0, \tag{44}$$

$$AX + XA^T - \kappa_2 X < 0, \tag{45}$$

$$QA + A^T Q - \kappa_2 Q < 0, \tag{46}$$

$$F_a(t_0, t) - \frac{\eta_1^*}{(\kappa_1+\kappa_2)\Delta_*} \le 0, \tag{47}$$

$$-\tau_a + \frac{\kappa_1+\kappa_2}{\kappa_1 - \eta_1^*} < 0, \tag{48}$$

where

$$\mathcal{B} = \frac{1}{\sigma}\ln\left(1 + \frac{d}{2\|L\|}\right),$$

$$\Upsilon = QA + A^T Q - YC - (YC)^T + \kappa_1 Q + (\gamma^2+1)I_n$$

with

$$\sigma = \|W\| + Nd\|M\|, \quad d = \frac{\gamma_2}{N}, \; Y = QG.$$

Further, the controller gain and observer gain are calculated by

$$K = -\frac{B^T X^{-1}}{2}, \qquad G = Q^{-1} Y. \tag{49}$$

Proof: Note that, taking Λ and $\mathcal{P}$ defined in (20) into (16), we can rewritten (16) in the following form

$$\begin{bmatrix} \Theta + \gamma^2 (I_N \otimes I_n) & 0 \\ +\kappa_1 \Xi \otimes P & \Pi + I_N \otimes \\ * & (\gamma^2 I_n + \kappa_1 Q) \end{bmatrix} < 0. \tag{50}$$

By using the separation principle, it obviously can be seen that if

$$\Theta + \gamma^2 (I_N \otimes I_n) + \kappa_1 \Xi \otimes P < 0 \tag{51}$$

and

$$\Pi + I_N \otimes (\gamma^2 I_n + \kappa_1 Q) < 0, \tag{52}$$

then (50) will be guaranteed, namely, (16) can be guaranteed.

Define $X = P^{-1}$, then taking Θ defined in (21) into (51) and pre- and post-multiplying inequality (51) by the matrix $I_N \otimes \mathrm{diag}\{X, X\}$, we can obtain (51) by using the Schur Lemma as equivalent to

$$\begin{bmatrix} \Xi \otimes (AX + XA^T + \kappa_1 X & -\frac{1}{2} c \Xi \otimes BB^T \\ -\mathrm{ca}(L) BB^T) + \gamma^2 (I_N \otimes XX) & \\ * & -\Xi \otimes (\lambda_{\max}(LL^T) + 1)^{-1} XX \end{bmatrix} < 0. \tag{53}$$

Further, (53) can be decomposed into

$$\begin{bmatrix} \Xi \otimes (AX + XA^T & \\ +\kappa_1 X - \mathrm{ca}(L) BB^T) & -\frac{1}{2} c \Xi \otimes BB^T \\ +I_N \otimes (\varepsilon_1 X) & \\ * & -\Xi \otimes (\varepsilon_2 X) \end{bmatrix} + \begin{bmatrix} -I_N \otimes (\varepsilon_1 X) & 0 \\ +\gamma^2 (I_N \otimes XX) & \\ & \Xi \otimes (\varepsilon_2 X \\ * & -(\lambda_{\max}(LL^T) \\ & +1)^{-1} XX) \end{bmatrix} < 0. \tag{54}$$

From (54) we know that if (42) and the following inequality hold

$$\begin{bmatrix} -I_N \otimes (\varepsilon_1 X) & 0 \\ +\gamma^2 (I_N \otimes XX) & \\ & \Xi \otimes (\varepsilon_2 \\ * & X - (\lambda_{\max} \\ & (LL^T) + 1)^{-1} XX) \end{bmatrix} < 0, \tag{55}$$

then (53) will be guaranteed, and further (51) holds.

In the following, in order to guarantee that (55) holds, post-multiplying (43) by the matrix $I_N \otimes \mathrm{diag}\{X, X\}$, one can obtain (55) and the specific proof is given in Remark 3.2. So we can obtain that if (42) and (43) hold, then (53) holds, further (51) holds orderly. Next, defining $Y = QG$ and taking Π defined in (20) into (52), we can obtain that (44) is equivalent to (52).

Combing the above proof, if we can guarantee that(42) –(44) hold, then (51) and (52) will hold. So further we can obtain that (50) holds means (16) also holds.

On the other hand, taking $\bar{W}$ defined in (15) and $\mathcal{P}$ defined in (20) into (17), we can rewrite (17) as follows:

$$\begin{bmatrix} \Xi \otimes (PA + A^T P & 0 \\ -\kappa_2 P) & \\ * & I_N \otimes (QA + \\ & A^T Q - \kappa_2 Q) \end{bmatrix} < 0. \tag{56}$$

Similarly, taking X into (56) and using the separation principle, we know that (45) and (46) holding can guarantee (56) holding. Therefore, (17) holds. Until now, we can obtain that LMIs (42)–(46) holding can guarantee (16) and (17) holding. In addition, the controller gain and the observer gain can be calculated as $K = -\frac{1}{2} B^T X$ and $G = Q^{-1} Y$, respectively.

So, observer-based controller (6) with gains (49) solves the OBSCP. ■

Remark 3.2: In Theorem 3.2, the separation method is employed to design the observer-based controller, which results in more general convex design conditions compared with H. Zhang et al. (2014) and Ruan et al. (2020).

Remark 3.3: It is worth noting that post-multiplying (43) by the matrix $I_N \otimes \mathrm{diag}\{X, X\}$, the direction of the inequality sign of (43) remains unchanged. According to

$$-\begin{bmatrix} I_N \otimes (-\varepsilon_1 I_n + \gamma^2 X) & 0 \\ * & \Xi \otimes \left(\varepsilon_2 I_n - \frac{1}{(\lambda_{\max}(LL^T) + 1)} X \right) \end{bmatrix} > 0$$

and $I_N \otimes \mathrm{diag}\{X, X\} > 0$. There are invertible matrices P and Q such that

$$-\begin{bmatrix} I_N \otimes (-\varepsilon_1 I_n + \gamma^2 X) & 0 \\ * & \Xi \otimes \left(\varepsilon_2 I_n - \frac{1}{(\lambda_{\max}(LL^T) + 1)} X \right) \end{bmatrix} = P^T P$$

and $I_N \otimes \mathrm{diag}\{X, X\} = Q^TQ$. Therefore,

$$-\begin{bmatrix} I_N \otimes (-\varepsilon_1 I_n + \gamma^2 X) & 0 \\ * & \Xi \otimes \left(\varepsilon_2 I_n - \frac{1}{(\lambda_{\max}(LL^T)+1)}X\right) \end{bmatrix} \times \begin{bmatrix} X & 0 \\ * & X \end{bmatrix} = P^TPQ^TQ.$$

Then,

$$Q\left(-\begin{bmatrix} I_N \otimes (-\varepsilon_1 I_n + \gamma^2 X) & 0 \\ * & \Xi \otimes \left(\varepsilon_2 I_n - \frac{1}{(\lambda_{\max}(LL^T)+1)}X\right) \end{bmatrix} \times \begin{bmatrix} X & 0 \\ * & X \end{bmatrix}\right) Q^{-1} = QP^TPQ^TQQ^{-1} = (PQ)^TPQ$$

is a positive definite matrix, and similar to

$$-\begin{bmatrix} I_N \otimes (-\varepsilon_1 I_n + \gamma^2 X) & 0 \\ * & \Xi \otimes \left(\varepsilon_2 I_n - \frac{1}{(\lambda_{\max}(LL^T)+1)}X\right) \end{bmatrix} \times \begin{bmatrix} X & 0 \\ * & X \end{bmatrix}.$$

That is,

$$-\begin{bmatrix} I_N \otimes (-\varepsilon_1 I_n + \gamma^2 X) & 0 \\ * & \Xi \otimes \left(\varepsilon_2 I_n - \frac{1}{(\lambda_{\max}(LL^T)+1)}X\right) \end{bmatrix} \times \begin{bmatrix} X & 0 \\ * & X \end{bmatrix} = -\begin{bmatrix} -I_N \otimes (\varepsilon_1 X) + \gamma^2 (I_N \otimes XX) & 0 \\ * & \Xi \otimes (\varepsilon_2 X - (\lambda_{\max}(LL^T) + 1)^{-1} XX) \end{bmatrix} > 0,$$

which is equivalent to (55).

4. Simulation

In order to show the effectiveness of the derived results, a comparative example simulation was performed. The simulation results show that the designed control scheme is resilient to DoS attacks.

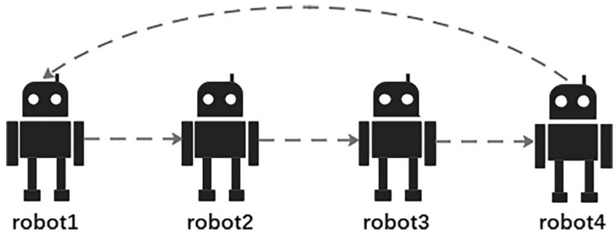

Figure 2. The communication topology $\mathcal{G}$.

A multirobot system illustrated in Feng and Hu (2019) described by (2) is given as

$$A = \begin{bmatrix} 0 & -0.5 \\ 0.5 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

The control objective of the simulation is to make all the robots reach a consensus on their positions x_{i1} and speeds x_{i2}, $i = 1, 2, 3, 4$, respectively. The communication between robots is described by the topology shown in Figure 2. The Laplacian matrix of the topology is given as

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix}.$$

In the following, the resilient control scheme designed by Theorem 3.2 reflects resilience to DoS attacks by comparing it with a conventional control scheme designed without considering the impact of DoS. Choose the event-triggered thresholds as $\beta_1 = 0.030$, $\beta_2 = 0.028$, $\beta_3 = 0.029$, $\beta_4 = 0.030$, the scalars $\kappa_1 = 0.077$, $\kappa_2 = 0.240$, $\varepsilon_1 = 0.04$, $\varepsilon_2 = 0.50$, $\eta_1^* = 0.0053$, $\gamma_1 = 0.031$, $\gamma_2 = 0.069$ and $\gamma = 0.1$, then, based on (47) and (48), we have $F_a(t_0, t) \leq 0.1672$ and $\tau_a \geq 4.4212$. This means that DoS attacks cannot exceed 0.1672 times per unit time according to (12). According to Theorem 3.2 and Lemma 2.1, we can obtain the controller gain as $K = \begin{bmatrix} 0.0365 & -0.1539 \end{bmatrix}$, the observer gain as $G = \begin{bmatrix} 0.3884 \\ -0.0611 \end{bmatrix}$, $c = 3.2487$ and a$(L) = 0.9998$.

Choosing b_i as 0.01s with the upper bound $\mathcal{B} = 0.0101$s, $\upsilon_i = 4$s, $\Xi_0 = 2.1$s and selecting the initial states as $x_1(0) = \begin{bmatrix} 1.5 \\ 0.35 \end{bmatrix}$, $x_2(0) = \begin{bmatrix} 1.3 \\ -0.5 \end{bmatrix}$, $x_3(0) = \begin{bmatrix} -1.3 \\ -1.2 \end{bmatrix}$ and $x_4(0) = \begin{bmatrix} 1.4 \\ -1.35 \end{bmatrix}$, respectively, then Figure 3 plots the position and speed evolutions of all robots under DoS attacks by using the resilient control scheme. We can see that the velocity and position states of each robot converge in finite time. The consensus errors $\delta_i(t)$ and $\hat{\delta}_i(t)$ of a multirobot system using the resilient control scheme and the traditional control scheme are given in Figures 4 and 5, respectively. The simulation results show that the resilient control strategy designed with Theorem 3.2 can accomplish the consensus control objective of the multi-robot system faster than the traditional control strategy that does not consider the effect of DoS attacks. In addition, Figure 6 shows the

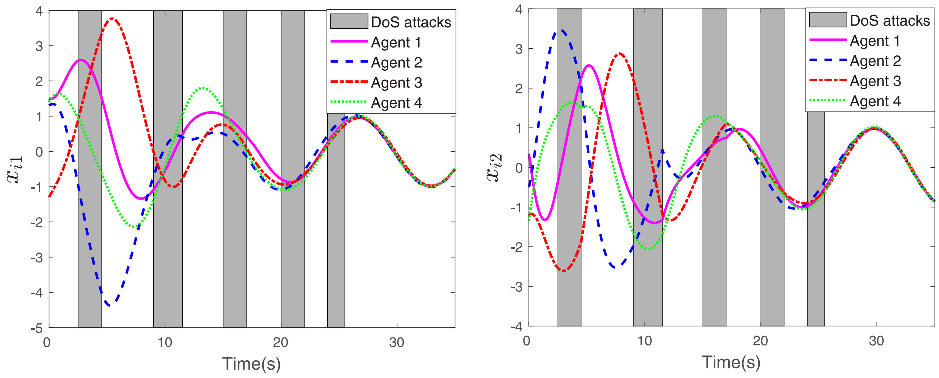

Figure 3. Trajectories of positions x_{i1} (left) and speeds x_{i2} (right) with the resilient control scheme.

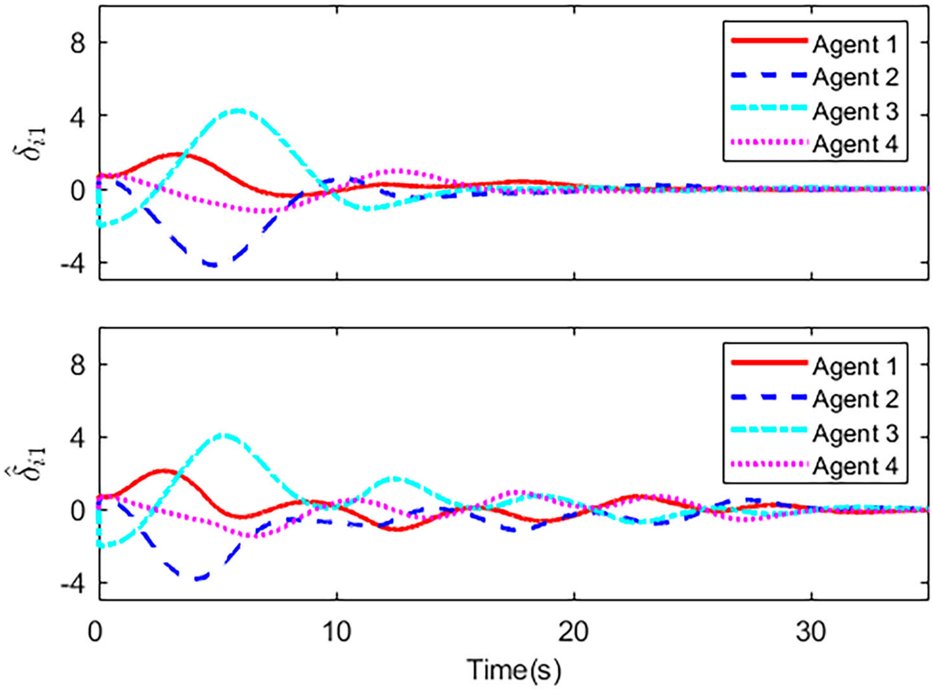

Figure 4. Consensus error $\delta_{i1}(t)$ with the resilient control scheme (up) and $\hat{\delta}_{i1}(t)$ with the traditional control scheme (down) of each UVAs ($i = 1, 2, 3, 4$).

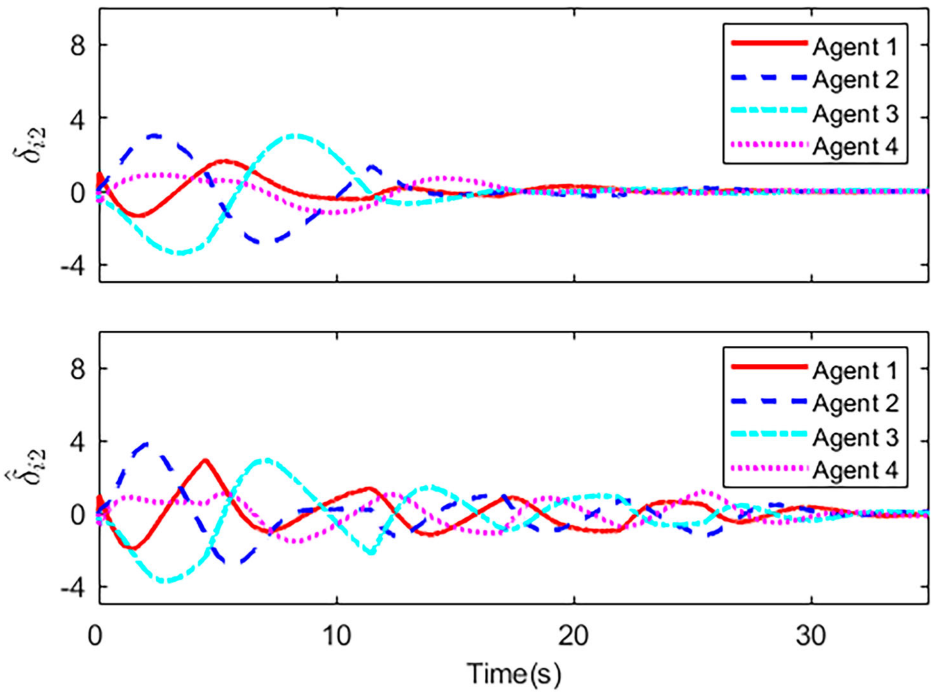

Figure 5. Consensus error $\delta_{i2}(t)$ with the resilient control scheme (up) and $\hat{\delta}_{i2}(t)$ with the traditional control scheme (down) of each UVAs ($i = 1, 2, 3, 4$).

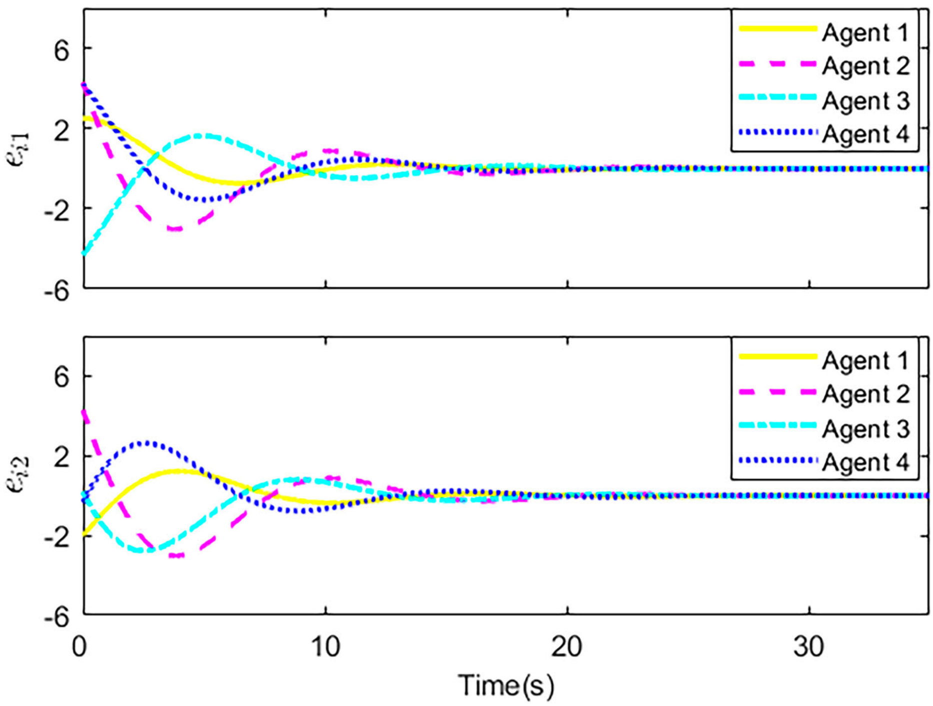

Figure 6. Trajectories of state estimation errors $e_i(t)$ of robots with the resilient control scheme.

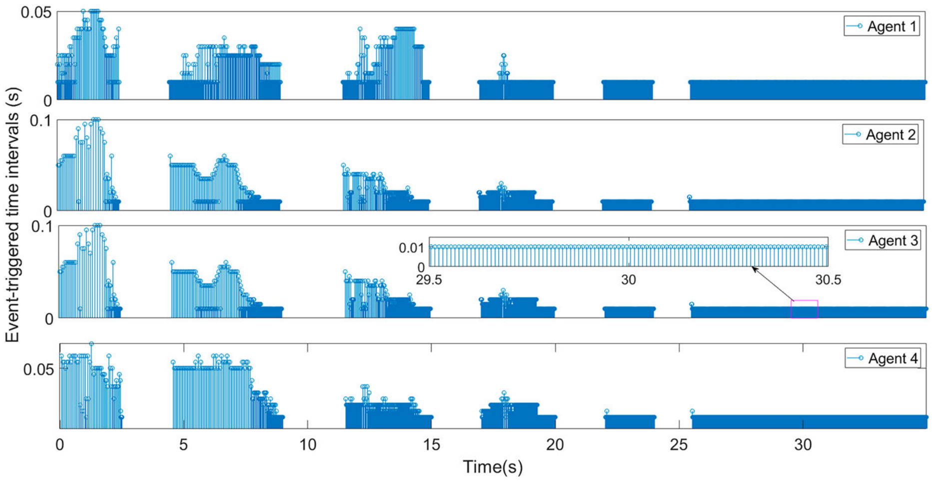

Figure 7. The event-triggered time intervals for the ith robot ($i = 1, 2, 3, 4$) with the resilient control scheme.

state estimation error of each agent. Figure 7 depicts the time intervals between events for all agents, and event-triggered mechanism fails when DoS attack occurs. We can see that there is no Zeno behaviour in the designed event-triggered mechanism. It can be observed that the consensus control objective of the distributed multi-robot system is obtained even with DoS attacks occurring.

From the example, we can see that the proposed observer-based event-triggered controller can achieve the leaderless consensus under DoS attacks and the Zeno behaviour is eliminated, which shows the effectiveness of the proposed method.

5. Conclusions

This paper considers the event-triggered consensus control problem for general linear MASs with the directed graph under DoS attacks. An observer-based event-triggered controller is proposed and the Zeno behaviour is eliminated with the help of the proposed event-triggered condition. And the observer-based controller design conditions are converted into convex ones by using the Separation Principle. Simulation results show that the objective of the state consensus with the proposed event-triggered controller can be achieved even when the DoS attacks occur.

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