Adaptive Output Feedback Funnel Control of Uncertain Nonlinear Systems With Arbitrary Relative Degree

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***Abstract*—This article considers the problem of output feedback funnel control for a class of uncertain nonlinear systems with arbitrary known relative degree. By utilizing the funnel control approach with a barrier Lyapunov function, a novel adaptive output feedback controller is constructed recursively, which accomplishes the output tracking with prescribed transient behavior. The developed control scheme features that the precise knowledge of system nonlinearities, including generally required bounding functions, is not needed. A physical example is performed to verify the effectiveness of the proposed theoretical findings.**

***Index Terms*—Adaptive control, barrier Lyapunov function, funnel control, uncertain nonlinear systems.**

I. Introduction

Driven by practical significance and theoretical challenge, tracking control with prescribed transient behavior for uncertain systems has become an active research topic in the past decades. In [1] and [2], adaptive tracking control methods with prescribed transient behavior were reported for classes of linear plants. Motivated by these results, two approaches were parallelly proposed, i.e., funnel control and prescribed performance control (PPC), to deal with the nonlinear paradigm. For details, the reader may refer to [3]–[12] and references therein. In spite of tremendous progress using states as feedback signals, the development of output feedback control with prescribed transient behavior for nonlinear systems still constitutes a challenging task. Within backstepping and a filter/precompensator, such a challenge was first handled in [13] using funnel control approach, leading to, however, a complicated solution. A relaxed controller was recently developed in [14] using high-gain observers. However, unlike [13], the resulted algorithm requires the precise knowledge of the control coefficient. As an alternative, adaptive output feedback PPC schemes were developed in conjunction with neural and fuzzy techniques for uncertain nonlinear systems [15], [16], but only guaranteeing semiglobal stability. To relax the use of approximation structures, approximation-free output feedback PPC frameworks were proposed for single-input-single-output (SISO) and multi-input multi-output (MIMO) nonlinear systems [17], [18], respectively. Nevertheless, unlike with the funnel controllers [13], [14], the PPC technique suffers from the problem of initial conditions, i.e., the choices of the performance functions in the PPC designs highly rely on the initial conditions, which may limit the practicability of the proposed controller.

Motivated by the above observations, this article concentrates on the problem of adaptive output feedback funnel control for a class of uncertain nonlinear systems with arbitrary known relative degree. By utilizing the funnel control approach with a barrier Lyapunov function, an adaptive output feedback controller is designed recursively, where the output tracking error can be steered to a prespecified strict error bound guaranteeing a desired transient response as well as a desired arbitrary tracking accuracy [5]. Different from the existing output feedback funnel control or PPC schemes, we will use a barrier Lyapunov function to cope with the system containing unknown output nonlinear functions or slow time-varying parameters, which implies that the proposed adaptive funnel controller does not incorporate *any prior knowledge* of system nonlinearities. One potential benefit of the proposed method is that it can avoid the problem of initial conditions in PPC design. Finally, a physical example is provided to illustrate the effectiveness of the proposed control method.

Notations: R, $R_{\geq 0}$, $R_{>0}$ denote the sets of real numbers, nonnegative real numbers, and positive real numbers, respectively. R^n denotes the real n-dimensional space. I_n stands for the $n \times n$ identity matrix. $\lambda_{\max}(U)$ denotes the maximum eigenvalue of the matrix U. $\mathcal{W}^{r,\infty}(R_{\geq 0} \to R)$ represents the set of r-times continuously differentiable function $f : R_{\geq 0} \to R$ with $f, \dot{f}, \ldots, f^{(r)}$ being essentially bounded on $R_{\geq 0}$. $f|_X$ denotes restriction of the function $f : R \to R$ to $X \subseteq R$.

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II. Preliminaries and Problem Description

In this section, we will present some essential concepts and technical lemmas to promote the controller design, and then, give the system description.

A. Preliminaries

Consider the initial value problem

$$\dot{\eta}_d(t) = h_d(t, \eta_d(t)), \quad \eta_d(0) = \eta_d^0 \in \Omega_d \tag{1}$$

where $h_d : R_{\geq 0} \times \Omega_d \to R^p$ is piecewise continuous in t and locally Lipschitz in η_d on $R_{\geq 0} \times \Omega_d$, and $\Omega_d \subseteq R^p$ is a nonempty open set.

Definition 1 (see [19]): The solution of (1) is said to be maximal, if it has no proper right extension that is also a solution of (1).

Definition 2 (see [20]): The solution of (1) is said to be *globally uniformly bounded* if for an arbitrarily large a, there is $\kappa = \kappa(a)$, independent of $t_0 \geq 0$, such that

$$||\eta_d(t_0)|| \leq a \Rightarrow ||\eta_d(t)|| \leq \kappa, \ \ \forall t \geq t_0. \tag{2}$$

Lemma 1 (see [19]): For each $\eta_d^0 \in \Omega_d$, there exists a unique maximal solution $\eta_d : [0, \tau'_{\max}) \to \Omega_d$ of (1) on the time interval $[0, \tau'_{\max})$ with $\tau'_{\max} \in R_{>0}$.

Lemma 2 (see [20]): Let $\eta_d : [0, \tau'_{\max}) \to \Omega_d$ be the maximal solution of (1) on the time interval $[0, \tau'_{\max})$ with $\tau'_{\max} < +\infty$, and Ω'_d be a compact subset of Ω_d, then, there exists $t'_s \in [0, \tau'_{\max})$ such that $\eta_d(t'_s) \notin \Omega'_d$.

Lemma 3 (see [21]): The following inequality holds for any $z \in R$ satisfying $|z| < 1$:

$$\log \frac{1}{1 - z^2} \leq \frac{z^2}{1 - z^2}. \tag{3}$$

B. Problem Description

Consider a class of SISO nonlinear system in the following form:

$$\begin{aligned} \dot{x} &= Ax + \varphi(t, y) + bu \\ y &= x_1 \end{aligned} \tag{4}$$

where

$$A = \begin{bmatrix} 0 & & \\ \vdots & I_{n-1} & \\ 0 & \cdots & 0 \end{bmatrix}, \varphi(t, y) = \begin{bmatrix} \varphi_1(t, y) \\ \vdots \\ \varphi_n(t, y) \end{bmatrix}$$

$b = [0, \ldots, 0, b_m, \ldots, b_0]^T \in R^n, x = [x_1, \ldots, x_n]^T \in R^n$ is the state vector, $u \in R$ is the control input, $y \in R$ is the system output with the initial condition $y(0)$. The system nonlinearities $\varphi_i(t, y) \in R$, $i = 1, \ldots, n$, are piecewise continuous in t and locally Lipschitz in y, which are *not necessarily known*, $b_m, \ldots, b_0$ are unknown constants.

Remark 1: In fact, there are many practical systems which can be modeled by (4), such as a single-link flexible robot [22], the dynamics of a ship [23], and the nonlinear pendulum [24].

For the considered system (4), the following assumptions are imposed.

Assumption 1: The sign of b_m and the relative degree $\rho = n - m$ are known.

Assumption 2: The unknown system nonlinearities $\varphi_i(t, y)$, $i = 1, \ldots, n$ satisfy

$$|\varphi_i(t, y)| \leq \bar{\varphi}_i(y), \ \ i = 1, \ldots, n \tag{5}$$

where $\bar{\varphi}_i(y), i = 1, \ldots, n$, are unknown nonnegative continuous functions.

Assumption 3: The system (4) is minimum phase, i.e., the polynomial $B(s) = b_m s^m + \cdots + b_1 s + b_0$ is Hurwitz.

Assumption 4: The desired trajectory y_r and its ρ order derivatives are known and bounded.

Remark 2: It is worth pointing out that Assumptions 1, 3, and 4 are quite standard for controller designs of system (4), which can be commonly found in literature, for example, [22], [23]. Assumption 2 implies that the analytical expressions of system nonlinearities $\varphi_i(t, y)$ and their bounding functions $\bar{\varphi}_i(y)$ are not demanded, in contrast to the common condition used in [22] and [23].

Based on the above assumptions, the control objective is to design an output feedback controller u for system (4) such that

1) all signals in the closed-loop system are globally bounded;
2) the tracking error $e = y - y_r$ evolves within a prescribed performance funnel

$$\mathcal{F}_\mu := \left\{(t, e) \in R_{\geq 0} \times R \middle| \mu|e| < 1\right\} \tag{6}$$

which is determined by a funnel function μ belonging to

$$\begin{aligned} \Phi := \Big\{ &\mu \in \mathcal{W}^{\rho,\infty}(R_{\geq 0} \to R) \Big| \mu(s) > 0, \forall s > 0 \\ &\text{and } \mu^{-1}|_{[\varepsilon,\infty)} \in \mathcal{W}^{\rho,\infty}([\varepsilon, \infty) \to R), \forall \varepsilon > 0 \Big\}. \end{aligned} \tag{7}$$

Remark 3: The output tracking with prescribed transient behavior can be satisfied by the reciprocal of μ. It is explicitly allowed that $\mu(0) = 0$, implying that the restriction on the initial value is relaxed due to $\mu(0)|e(0)| < 1$.

It should be mentioned that the output feedback controller satisfies the above requirement is socalled output feedback funnel control [13], [24].

III. Adaptive Output Feedback Funnel Control Design

In this section, we first design state filters to estimate the unmeasure states of system (4), then propose an adaptive output feedback funnel control scheme to achieve the prescribed transient behavior, and finally present the rigorous stability analysis via a barrier Lyapunov function.

A. State Estimation Filters

To begin with, the state estimation filters are designed using only the input u and output y as follows:

$$\dot{\xi} = A_0 \xi + ky \tag{8}$$

$$\dot{\lambda} = A_0 \lambda + e_n u \tag{9}$$

$$\upsilon_i = A_0^i \lambda, \ \ i = 0, 1, \ldots, m \tag{10}$$

where A_0^i denotes the ith power of the matrix A_0, e_i is the ith coordinate vector in R^n, i.e., $e_i = \Big[\underbrace{0, \ldots, 0}_{i-1}, 1, 0, \ldots, 0\Big]^T \in R^n$, $k = [k_1, \ldots, k_n]^T \in R^n$ is chosen such that $A_0 = A - ke_1^T$ is Hurwitz, i.e., $p(s) = s^n + k_1 s^{n-1} + \cdots + k_{n-1} s + k_n$ is a Hurwitz polynomial. With the designed *K*-filters, the state estimators are given by

$$\hat{x} = \xi + \sum_{i=0}^{m} b_i \upsilon_i. \tag{11}$$

Define $\epsilon = x - \hat{x}$, we obtain

$$\dot{\epsilon} = A_0 \epsilon + \varphi(t, y) \tag{12}$$

where

$$\epsilon = \begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_n \end{bmatrix}, \ \ A_0 = \begin{bmatrix} -k_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -k_{n-1} & 0 & \cdots & 1 \\ -k_n & 0 & \cdots & 0 \end{bmatrix}.$$

Note that A_0 is a Hurwitz matrix. Therefore, there exists a positive-definite matrix $P = P^T > 0$ such that

$$A_0^T P + P A_0 = -I. \tag{13}$$

Let $V_0(\epsilon) = \epsilon^T P\epsilon$. By Assumption 2, it is obtained that

$$\begin{aligned}\dot{V}_0 &= -||\epsilon||^2 + 2\epsilon^T P\varphi(t,y)\\ &\leq -||\epsilon||^2 + 2\lambda_{\max}(P)||\epsilon||\sum_{j=1}^{n}\bar{\varphi}_j(y)\\ &\leq -\frac{3}{4}||\epsilon||^2 + 4\left(\sum_{j=1}^{n}\lambda_{\max}(P)\bar{\varphi}_j(y)\right)^2. \end{aligned} \tag{14}$$

Now, the system (4) is expressed as

$$\dot{y} = b_m \upsilon_{m,2} + \xi_2 + \varphi_1(t,y) + \bar{w}^T\Theta + \epsilon_2 \tag{15}$$

$$\dot{\upsilon}_{m,i} = \upsilon_{m,i+1} - k_i\upsilon_{m,1}, \quad i = 2,\ldots,\rho-1 \tag{16}$$

$$\dot{\upsilon}_{m,\rho} = \upsilon_{m,\rho+1} - k_\rho\upsilon_{m,1} + u \tag{17}$$

where

$$w = [\upsilon_{m,2}, \upsilon_{m-1,2},\ldots,\upsilon_{0,2}]^T \tag{18}$$

$$\bar{w} = [0, \upsilon_{m-1,2},\ldots,\upsilon_{0,2}]^T \tag{19}$$

$$\Theta = [b_m,\ldots,b_0]^T \tag{20}$$

and $\upsilon_{i,2}$, ξ_2, ϵ_2 denote the second entries of υ_i, ξ, ϵ, respectively.

Remark 4: Different from the existing output feedback results in [22] and [23], this article only uses two filters (8) and (9) to estimate the states of system, which is shown in (11). It is noted that $\hat{x}$ is not available for control design owing to the unknown parameters $b_m,\ldots,b_0$. Instead, the *K*-filters (8) and (9) are employed directly to design the adaptive funnel controller (28) in the sequel.

B. Adaptive Output Feedback Funnel Controller

Initially, we select a funnel function μ that satisfies $\mu(0)|y(0) - y_r(0)| < 1$, and incorporates the desired performance specifications regarding the steady-state error and the speed of convergence. Let $z_1 = \mu(y - y_r)$ and $z_i = \upsilon_{m,i} - \hat{\varrho}y_r^{(i-1)} - \alpha_{i-1}$, $i = 2,\ldots,\rho$, where $\hat{\varrho}$ is the estimate of $\varrho = \frac{1}{b_m}$, the adaptive output feedback funnel controller, the tuning functions τ_i, and the adaptive laws are designed as follows:

$$\alpha_1 = \hat{\varrho}\bar{\alpha}_1 \tag{21}$$

$$\bar{\alpha}_1 = -\frac{c_1(y-y_r)}{1-z_1^2} - \xi_2 - \bar{w}^T\hat{\Theta} \tag{22}$$

$$\begin{aligned}\alpha_2 = &-\frac{\hat{b}_m\mu z_1}{(1-z_1^2)} - \left(c_2 + (d_2+\gamma_2)\left(\frac{\partial\alpha_1}{\partial y}\right)^2\right)z_2\\ &+ \beta_2 + \frac{\partial\alpha_1}{\partial\hat{\Theta}}\Gamma(\tau_2 - \sigma_\theta\hat{\Theta})\end{aligned} \tag{23}$$

$$\begin{aligned}\alpha_i = &-z_{i-1} - \left(c_i + (d_i+\gamma_i)\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2\right)z_i\\ &+ \beta_i + \frac{\partial\alpha_{i-1}}{\partial\hat{\Theta}}\Gamma(\tau_i - \sigma_\theta\hat{\Theta})\\ &- \left(\sum_{k=2}^{i-1} z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\Theta}}\right)\Gamma\frac{\partial\alpha_{i-1}}{\partial y}w, \quad i = 3,\ldots,\rho\end{aligned} \tag{24}$$

$$\begin{aligned}\beta_i = &\frac{\partial\alpha_{i-1}}{\partial y}(\xi_2 + w^T\hat{\Theta}) + k_i\upsilon_{m,1}\\ &+ \frac{\partial\alpha_{i-1}}{\partial\xi}(A_0\xi + ky) + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\zeta^{(j-1)}}\zeta^{(j)}\\ &+ \sum_{j=1}^{m+i-1}\frac{\partial\alpha_{i-1}}{\partial\lambda_j}(-k_j\lambda_1 + \lambda_{j+1})\\ &+ (y_r^{(i-1)} + \frac{\partial\alpha_{i-1}}{\partial\hat{\varrho}})\dot{\hat{\varrho}}, \quad i = 2,\ldots,\rho\end{aligned} \tag{25}$$

$$\tau_1 = \frac{\mu z_1}{(1-z_1^2)}(w - \hat{\varrho}(\dot{y}_r + \bar{\alpha}_1)e_1) \tag{26}$$

$$\tau_i = \tau_{i-1} - \frac{\partial\alpha_{i-1}}{\partial y}wz_i, \quad i = 2,\ldots,\rho \tag{27}$$

$$u = \alpha_\rho - \upsilon_{m,\rho+1} + \hat{\varrho}y_r^{(\rho)} \tag{28}$$

$$\dot{\hat{\varrho}} = -\gamma_\varrho\left[\mathrm{sign}(b_m)(\dot{y}_r + \bar{\alpha}_1)\frac{\mu z_1}{(1-z_1^2)} + \sigma_\varrho\hat{\varrho}\right] \tag{29}$$

$$\dot{\hat{\Theta}} = \Gamma(\tau_\rho - \sigma_\theta\hat{\Theta}) \tag{30}$$

where c_1, c_i, d_i, γ_i, γ_ϱ, σ_ϱ, and σ_θ are positive design constants, $i = 2,\ldots,\rho$, Γ is a positive definite design matrix, $\hat{\Theta}$ is the estimate of Θ, and $\zeta := [y_r, \mu]^T$.

Remark 5: As revealed in (21)–(30), the proposed adaptive funnel controller (28) does not incorporate *a priori knowledge* of system nonlinearities $\varphi_i(t,y)$ and their corresponding bounding functions $\bar{\varphi}_i(y)$, thus relaxing the assumption used in the output feedback control solutions [22], [23].

C. Stability Analysis

The main results of this article are summarized in the following theorem.

Theorem 1: Consider the closed-loop system consisting of the plant (4) under Assumptions 1–4, the *K*-filters (8) and (9), the output feedback controller (28), and the adaptive laws (29)–(30). Then, for any initial condition $y(0)$, the following statements hold:

1) all the closed-loop signals are globally bounded;
2) the output tracking error $y - y_r$ evolves within the prescribed performance funnel (6).

Proof: The proof consists of three parts. In *Part I*, the existence and uniqueness of a maximal solution of the considered closed-loop system on the time interval $[0, \tau_{\max})$ is analyzed. In *Part II*, we show that $\tau_{\max} = +\infty$ by contradiction. The control objective is achieved in *Part III*.

Part I: From (21), (23), (24), and (28), the virtual and actual control laws can be written as functions of the vector $\eta = [z_1, x_2,\ldots,x_n,\xi^T,\lambda^T,\hat{\varrho},\hat{\Theta}^T]^T$ as follows:

$$\begin{aligned}\alpha_1 &= \alpha_1(y, z_1, \xi, \bar{\lambda}_{m-1}, \hat{\varrho}, \hat{\Theta}, y_r)\\ &= \alpha_1^*(t, z_1, \xi^T, \bar{\lambda}_{m-1}^T, \hat{\varrho}, \hat{\Theta}^T)\end{aligned} \tag{31}$$

$$\begin{aligned}\alpha_i &= \alpha_i(y, z_1, \xi, \bar{\lambda}_{m+i}, \hat{\varrho}, \hat{\Theta}, \bar{y}_r^{(i-1)}, \bar{\mu}^{(i-1)})\\ &= \alpha_i^*(t, z_1, \xi^T, \bar{\lambda}_{m+i}^T, \hat{\varrho}, \hat{\Theta}^T), \quad i = 2,\ldots,\rho-1\end{aligned} \tag{32}$$

$$\begin{aligned}u &= u(y, z_1, \xi, \bar{\lambda}_n, \hat{\varrho}, \hat{\Theta}, \bar{y}_r^{(n-1)}, \bar{\mu}^{(n-1)})\\ &= u^*(t, z_1, \xi^T, \bar{\lambda}_n^T, \hat{\varrho}, \hat{\Theta}^T)\end{aligned} \tag{33}$$

where $\bar{\lambda}_i = [\lambda_1,\ldots,\lambda_i]^T$, $\bar{y}_r^{(i)} = [y_r,\ldots,y_r^{(i)}]$, $\bar{\mu}^{(i)} = [\mu,\ldots,\mu^{(i)}]$.

Consequently, the closed-loop system of the vector η is expressed as

$$\begin{aligned}\dot{z}_1 &= \mu(x_2 + \varphi_1(t,y) - \dot{y}_r) + \dot{\mu}(y - y_r)\\ &= h_1(t, x_2)\end{aligned} \tag{34}$$

$$\dot{x}_i = x_{i+1} + \varphi_i(t,y)$$

$$= h_i(t, x_{i+1}), \quad i = 2, \ldots, \rho - 1 \tag{35}$$

$$\begin{aligned}\dot{x}_{\rho+j} &= x_{\rho+j+1} + \varphi_{\rho+j}(t, y) + b_{m-j} u \\ &= h_{\rho+j}(t, z_1, x_{\rho+j+1}, \xi^T, \bar{\lambda}_n^T, \hat{\varrho}, \hat{\Theta}^T) \\ &\quad j = 0, \ldots, m\end{aligned} \tag{36}$$

$$\dot{\xi} = A_0 \xi + k y = \begin{bmatrix} h_{n+1}(t, \xi^T) \\ \vdots \\ h_{2n}(t, \xi^T) \end{bmatrix} \tag{37}$$

$$\dot{\lambda} = A_0 \lambda + e_n u = \begin{bmatrix} h_{2n+1}(\lambda^T) \\ \vdots \\ h_{3n-1}(\lambda^T) \\ h_{3n}(t, z_1, \xi^T, \bar{\lambda}_n^T, \hat{\varrho}, \hat{\Theta}^T) \end{bmatrix} \tag{38}$$

$$\begin{aligned}\dot{\hat{\varrho}} &= -\gamma_\varrho \left[\mathrm{sign}(b_m)(\dot{y}_r + \bar{\alpha}_1) \frac{\mu z_1}{(1 - z_1^2)} + \sigma_\varrho \hat{\varrho}\right] \\ &= h_{3n+1}(t, z_1, \xi^T, \bar{\lambda}_m^T, \hat{\varrho}, \hat{\Theta}^T)\end{aligned} \tag{39}$$

$$\dot{\hat{\Theta}} = \Gamma(\tau_\rho - \sigma_\theta \hat{\Theta}) = \begin{bmatrix} h_{3n+2}(t, z_1, \xi^T, \bar{\lambda}_n^T, \hat{\varrho}, \hat{\Theta}^T) \\ \vdots \\ h_{3n+m+2}(t, z_1, \xi^T, \bar{\lambda}_n^T, \hat{\varrho}, \hat{\Theta}^T) \end{bmatrix} \tag{40}$$

which can also be formulated as the following compact form:

$$\dot{\eta} = h(t, \eta) = \begin{bmatrix} h_1(t, \eta) \\ \vdots \\ h_{3n+m+2}(t, \eta) \end{bmatrix}. \tag{41}$$

Define the open set

$$\Omega = (-1, 1) \times R^{3n+m+1}. \tag{42}$$

Note that $|z_1(0)| < 1$, one observes $\eta(0) = [z_1(0), x_2(0), \ldots, x_n(0), \xi^T(0), \lambda^T(0), \hat{\varrho}(0), \hat{\Theta}^T(0)]^T \in \Omega$. Additionally, we can conclude that $h : R_{\geq 0} \times \Omega \to R^{3n+m+2}$ is piecewise continuous in t and locally Lipschitz in Ω, on $R_{\geq 0} \times \Omega$, owing to the fact that the desired trajectory y_r, the funnel function μ and their ρ order derivatives are piecewise continuous in t, and the functions $\varphi_i(t, y)$, $i = 1, \ldots, n$, are piecewise continuous in t and locally Lipschitz in y. Therefore, by Lemma 1, the conditions on h ensure the existence and uniqueness of a maximal solution $\eta(t)$ on the time interval $[0, \tau_{\max})$, i.e., $|z_1| < 1$ for all $t \in [0, \tau_{\max})$.

Part II: Next, to show $\tau_{\max} = +\infty$ by contradiction, we suppose that $\tau_{\max} < +\infty$.

For the system (41), consider the barrier Lyapunov function candidates V_i as follows [25]:

$$\begin{aligned}V_1 &= \frac{1}{2} \log \frac{1}{1 - z_1^2} + \frac{1}{2} \tilde{\Theta}^T \Gamma^{-1} \tilde{\Theta} \\ &\quad + \frac{|b_m|}{2\gamma_\varrho} \tilde{\varrho}^2 + \frac{\mu_{\max}^2}{c_1} V_0\end{aligned} \tag{43}$$

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{\gamma_i} V_0, \quad i = 2, \ldots, \rho \tag{44}$$

where $\tilde{\Theta} = \Theta - \hat{\Theta}$, $\tilde{\varrho} = \varrho - \hat{\varrho}$, $\mu_{\max} > 0$ is the upper bound of μ. Clearly, it is observed that V_i, $i = 1, \ldots, n$, are positive definite and continuously differentiable in the open set Ω.

Step 1: The time derivative of V_1 becomes

$$\begin{aligned}\dot{V}_1 &= \frac{z_1}{(1 - z_1^2)} \left[\mu \left(b_m z_2 + b_m \hat{\varrho}(\dot{y}_r + \bar{\alpha}_1) + \xi_2\right.\right. \\ &\quad \left.\left. + \varphi_1(t, y) + \bar{w}^T \Theta + \epsilon_2 - \dot{y}_r\right) + \dot{\mu}(y - y_r)\right] \\ &\quad - \tilde{\Theta}^T \Gamma^{-1} \dot{\hat{\Theta}} - \frac{|b_m|}{\gamma_\varrho} \tilde{\varrho} \dot{\hat{\varrho}} + \frac{\mu_{\max}^2}{c_1} \dot{V}_0 \\ &\leq -\frac{c_1 z_1^2}{(1 - z_1^2)^2} + \frac{\mu z_1 (b_m z_2 + \bar{w}^T \tilde{\Theta})}{(1 - z_1^2)} \\ &\quad + \frac{\mu |z_1| F_1}{(1 - z_1^2)} + \frac{\mu z_1 \epsilon_2}{(1 - z_1^2)} - \tilde{\Theta}^T \Gamma^{-1} \dot{\hat{\Theta}} \\ &\quad + |b_m| \sigma_\varrho \tilde{\varrho} \hat{\varrho} + \frac{4\mu_{\max}^2}{c_1} \left(\sum_{j=1}^n \lambda_{\max}(P) \bar{\varphi}_j(y)\right)^2 \\ &\quad - \frac{3\mu_{\max}^2}{4c_1} ||\epsilon||^2, \quad \forall t \in [0, \tau_{\max})\end{aligned} \tag{45}$$

where $F_1 = \mu|\bar{\varphi}_1(y)| + |\dot{\mu}(y - y_r)|$.

In view of (18), (20), and (21), it follows that:

$$b_m z_2 + \bar{w}^T \tilde{\Theta} = \hat{b}_m z_2 + (w - \hat{\varrho}(\dot{y}_r + \bar{\alpha}_1) e_1)^T \tilde{\Theta} \tag{46}$$

where $\hat{b}_m$ is the estimate of b_m, $\tilde{b}_m = b_m - \hat{b}_m$.

Using the fact that $\mu \leq \mu_{\max}$ and Young's inequality, it yields

$$\frac{\mu z_1}{(1 - z_1^2)} \epsilon_2 \leq \frac{c_1 z_1^2}{2(1 - z_1^2)^2} + \frac{\mu_{\max}^2}{2c_1} ||\epsilon||^2 \tag{47}$$

$$\frac{\mu |z_1| F_1}{(1 - z_1^2)} \leq \frac{c_1 z_1^2}{4(1 - z_1^2)^2} + \frac{\mu_{\max}^2}{c_1} F_1^2. \tag{48}$$

Based on (46)–(48), the expression in (45) reduces to

$$\begin{aligned}\dot{V}_1 &\leq -\frac{c_1 z_1^2}{4(1 - z_1^2)^2} - \frac{\mu_{\max}^2}{4c_1} ||\epsilon||^2 \\ &\quad + \tilde{\Theta}^T (\tau_1 - \Gamma^{-1} \dot{\hat{\Theta}}) + |b_m| \sigma_\varrho \tilde{\varrho} \hat{\varrho} \\ &\quad + \frac{\hat{b}_m \mu z_1 z_2}{(1 - z_1^2)} + \Delta_1, \quad \forall t \in [0, \tau_{\max})\end{aligned} \tag{49}$$

where $\Delta_1 = \frac{\mu_{\max}^2}{c_1} (F_1^2 + 4(\sum_{j=1}^n \lambda_{\max}(P) \bar{\varphi}_j(y))^2)$.

Noting the fact that $|z_1| < 1$ for all $t \in [0, \tau_{\max})$ from *Part I*, we have $\frac{1}{1 - z_1^2} \geq 1$ for all $t \in [0, \tau_{\max})$. Then, one gets

$$\begin{aligned}\dot{V}_1 &\leq -\frac{c_1 z_1^2}{4(1 - z_1^2)} - \frac{\mu_{\max}^2}{4c_1} ||\epsilon||^2 \\ &\quad + \tilde{\Theta}^T (\tau_1 - \Gamma^{-1} \dot{\hat{\Theta}}) + |b_m| \sigma_\varrho \tilde{\varrho} \hat{\varrho} \\ &\quad + \frac{\hat{b}_m z_1 z_2}{\mu(1 - z_1^2)} + \Delta_1, \quad \forall t \in [0, \tau_{\max}).\end{aligned} \tag{50}$$

Step i $(i = 2, \ldots, \rho - 1)$: From (44), the time derivative of V_i is given by

$$\begin{aligned}\dot{V}_i &= \dot{V}_{i-1} + z_i \Big(\upsilon_{m,i+1} - k_i \upsilon_{m,1} - \hat{\varrho} y_r^i - \dot{\hat{\varrho}} y_r^{i-1} \\ &\quad - \frac{\partial \alpha_{i-1}}{\partial y} (\xi_2 + \varphi_1(t, y) + w^T \Theta + \epsilon_2)\end{aligned}$$

$$
\begin{aligned}
&- \frac{\partial \alpha_{i-1}}{\partial \xi}(A_0\xi + ky) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \zeta^{(j-1)}}\zeta^{(j)} \\
&- \sum_{j=1}^{m+i-1} \frac{\partial \alpha_{i-1}}{\partial \lambda_j}(-k_j\lambda_1 + \lambda_{j+1}) - \frac{\partial \alpha_{i-1}}{\partial \hat{\Theta}}\dot{\hat{\Theta}} \\
&- \left.\frac{\partial \alpha_{i-1}}{\partial \hat{\varrho}}\dot{\hat{\varrho}}\right) + \frac{1}{\gamma_i}\dot{V}_0, \quad \forall t \in [0, \tau_{\max}). \qquad (51)
\end{aligned}
$$

Applying Assumption 2 and Young's inequality, we achieve

$$ -z_i \frac{\partial \alpha_{i-1}}{\partial y}\varphi_1(t,y) \le d_i \left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2 z_i^2 + \frac{1}{4d_i}\bar{\varphi}_1^2(y) \qquad (52) $$

$$ -z_i \frac{\partial \alpha_{i-1}}{\partial y}\epsilon_2 \le \gamma_i \left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2 z_i^2 + \frac{1}{4\gamma_i}||\epsilon||^2. \qquad (53) $$

From (23)–(25) and (27), one obtains

$$
\begin{aligned}
\dot{V}_i \le &- \frac{c_1 z_1^2}{4(1-z_1^2)} - \sum_{j=2}^{i} c_j z_j^2 - \frac{\mu_{\max}^2}{4c_1}||\epsilon||^2 \\
&- \sum_{j=2}^{i} \frac{1}{2\gamma_j}||\epsilon||^2 + \tilde{\Theta}^T(\tau_i - \Gamma^{-1}\dot{\hat{\Theta}}) \\
&+ \left(\sum_{k=2}^{i} z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\Theta}}\right)(\Gamma\tau_i - \Gamma\sigma_\theta\hat{\Theta} - \dot{\hat{\Theta}}) \\
&+ |b_m|\sigma_\varrho\tilde{\varrho}\hat{\varrho} + z_i z_{i+1} + \Delta_i, \quad \forall t \in [0, \tau_{\max}) \qquad (54)
\end{aligned}
$$

where $\Delta_i = \Delta_{i-1} + \frac{1}{4d_i}\bar{\varphi}_1^2(y) + \frac{4}{\gamma_i}(\sum_{j=1}^n \lambda_{\max}(P)\bar{\varphi}_j(y))^2$.

Step ρ: With the help of (17), it follows that

$$
\begin{aligned}
\dot{V}_\rho = \dot{V}_{\rho-1} + z_n \Big(&\upsilon_{m,\rho+1} - k_\rho \upsilon_{m,1} + u - \hat{\varrho} y_r^{\rho} - \dot{\hat{\varrho}} y_r^{\rho-1} \\
&- \frac{\partial \alpha_{\rho-1}}{\partial y}(\xi_2 + \varphi_1(t,y) + w^T\Theta + \epsilon_2) \\
&- \frac{\partial \alpha_{\rho-1}}{\partial \xi}(A_0\xi + ky) - \sum_{j=1}^{\rho-1} \frac{\partial \alpha_{\rho-1}}{\partial \zeta^{(j-1)}}\zeta^{(j)} \\
&- \sum_{j=1}^{m+\rho-1} \frac{\partial \alpha_{\rho-1}}{\partial \lambda_j}(-k_j\lambda_1 + \lambda_{j+1}) - \frac{\partial \alpha_{\rho-1}}{\partial \hat{\Theta}}\dot{\hat{\Theta}} \\
&- \left.\frac{\partial \alpha_{\rho-1}}{\partial \hat{\varrho}}\dot{\hat{\varrho}}\right) + \frac{1}{\gamma_\rho}\dot{V}_0, \quad \forall t \in [0, \tau_{\max}). \qquad (55)
\end{aligned}
$$

In view of (24), (25), (28), and (30), one gets

$$
\begin{aligned}
\dot{V}_\rho \le &- \frac{c_1 z_1^2}{4(1-z_1^2)} - \sum_{j=2}^{\rho} c_j z_j^2 \\
&- \frac{\mu_{\max}^2}{4c_1}||\epsilon||^2 - \sum_{j=2}^{\rho} \frac{1}{2\gamma_j}||\epsilon||^2 + \sigma_\theta\tilde{\Theta}^T\hat{\Theta} \\
&+ |b_m|\sigma_\varrho\tilde{\varrho}\hat{\varrho} + \Delta_n, \quad \forall t \in [0, \tau_{\max}) \qquad (56)
\end{aligned}
$$

where $\Delta_\rho = \Delta_{\rho-1} + \frac{1}{4d_\varrho}\bar{\varphi}_1^2(y) + \frac{4}{\gamma_\varrho}(\sum_{j=1}^n \lambda_{\max}(P)\bar{\varphi}_j(y))^2$.

Using Young's inequality, we obtain

$$ \sigma_\theta\tilde{\Theta}^T\hat{\Theta} \le -\frac{\sigma_\theta}{2}\tilde{\Theta}^T\tilde{\Theta} + \frac{\sigma_\theta}{2}\Theta^T\Theta \qquad (57) $$

$$ |b_m|\sigma_\varrho\tilde{\varrho}\hat{\varrho} \le -\frac{|b_m|\sigma_\varrho}{2}\tilde{\varrho}^2 + \frac{|b_m|\sigma_\varrho}{2}\varrho^2. \qquad (58) $$

Moreover, from *Part I*, it is immediate to obtain that y is bounded, since $|z_1| < 1$ for all $t \in [0, \tau_{\max})$, i.e., $\mu|y - y_r| < 1, \forall t \in [0, \tau_{\max})$. By utilizing the fact that μ, $\dot{\mu}$, y_r, y are bounded for all $t \in [0, \tau_{\max})$, and employing the Extreme Value Theorem owing to the continuity of $\bar{\varphi}_i(y)$, $i = 1, \ldots, n$, it leads to

$$ 0 \le \Delta_n \le \Delta_{\rho\max}, \quad \forall t \in [0, \tau_{\max}) \qquad (59) $$

where $\Delta_{\rho\max}$ is an unknown nonnegative constant.

Therefore

$$
\begin{aligned}
\dot{V}_\rho \le &- \frac{c_1 z_1^2}{4(1-z_1^2)} - \sum_{j=2}^{\rho} c_j z_j^2 \\
&- \frac{\mu_{\max}^2}{4c_1}||\epsilon||^2 - \sum_{j=2}^{\rho} \frac{1}{2\gamma_j}||\epsilon||^2 - \frac{\sigma_\theta}{2}\tilde{\Theta}^T\tilde{\Theta} \\
&- \frac{|b_m|\sigma_\varrho}{2}\tilde{\varrho}^2 + \Delta, \quad \forall t \in [0, \tau_{\max}) \qquad (60)
\end{aligned}
$$

where $\Delta = \Delta_{\rho\max} + \frac{\sigma_\theta}{2}\Theta^T\Theta + \frac{|b_m|\sigma_\varrho}{2}\varrho^2$.

By Lemma 3, we get

$$
\begin{aligned}
\dot{V}_\rho \le &- \frac{c_1}{4}\log\frac{1}{1-z_1^2} - \sum_{j=2}^{\rho} c_j z_j^2 \\
&- \frac{\mu_{\max}^2}{4c_1}||\epsilon||^2 - \sum_{j=2}^{\rho} \frac{1}{2\gamma_j}||\epsilon||^2 \\
&- \frac{\sigma_\theta}{2}\tilde{\Theta}^T\tilde{\Theta} - \frac{|b_m|\sigma_\varrho}{2}\tilde{\varrho}^2 + \Delta \\
\le &- \chi V_\rho + \Delta, \quad \forall t \in [0, \tau_{\max}) \qquad (61)
\end{aligned}
$$

where $\chi = \min\{\frac{c_1}{2}, 2c_2, \ldots, 2c_\rho, \frac{\sigma_\theta}{\lambda_{\max}(\Gamma^{-1})}, \gamma_\varrho\sigma_\varrho, \frac{1}{4\lambda_{\max}(P)}\}$.

Owing to $\dot{V}_\rho \le 0$ for all $V_\rho \ge \frac{\Delta}{\chi}$, (61) satisfies

$$ V_\rho \le \varpi = \max\left\{V_\rho(0), \frac{\Delta}{\chi}\right\}, \quad \forall t \in [0, \tau_{\max}). \qquad (62) $$

From (62), it is indicated that for all $t \in [0, \tau_{\max})$

$$ |z_1| \le \sqrt{1 - e^{-2\varpi}} < 1 \qquad (63) $$

and $z_2, \ldots, z_\rho$, $\hat{\Theta}$, $\hat{\varrho}$, ϵ are bounded.

By virtue of (8) and the fact that y and y_r are bounded for all $t \in [0, \tau_{\max})$, it follows that ξ is bounded for all $t \in [0, \tau_{\max})$ as A_0 is Hurwitz. Assumption 3 and (9) imply that $\lambda_1, \ldots, \lambda_{m+1}$ are bounded for all $t \in [0, \tau_{\max})$. Then, following the similar analysis to [22], for all $t \in [0, \tau_{\max})$, the boundedness of $\lambda_{m+2}, \ldots, \lambda_n$ are established. Finally, in view of (11) and the fact that ξ, λ, and ϵ are bounded for all $t \in [0, \tau_{\max})$, we can conclude that x is bounded for all $t \in [0, \tau_{\max})$. Furthermore, from (28), u is also bounded for all $t \in [0, \tau_{\max})$.

From above, it can wrap up that there exists a compact subset $\Omega' \subset \Omega$ such that the maximal solution of (41) satisfies $\eta(t) \in \Omega'$ for all $t \in [0, \tau_{\max})$. By Lemma 2, we can conclude by contradiction that $\tau_{\max} = +\infty$, i.e., $\eta(t)$ is defined for all $t \in [0, +\infty)$.

Part III: To repeat the procedures from $Step$ 1–$Step$ ρ in *Part II*, it is deduced progressively that all the signals of the closed-loop system (41) are bounded for all $t \in [0, +\infty)$. According to Definition 2, we can conclude that all the closed-loop signals are globally uniformly bounded. Moreover, $|z_1| < 1$ leads to

$$ \mu|x_1 - y_r| < 1, \quad \forall t \ge 0 \qquad (64) $$

which completes the proof. ■

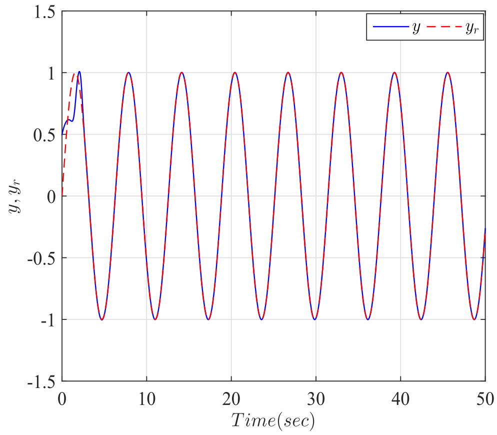

Fig. 1. y and y_r (rad/s).

Remark 6: The barrier function $\frac{1}{2}\log\frac{1}{1-z_1^2}$ shown in (43) and (44) plays a crucial role to cope with the unknown system nonlinearities and ensure the output tracking error with prescribed transient behavior.

Remark 7: The utilization of barrier Lyapunov function for adaptive control problems was extensively studied in [21], and [25]–[27] for classes of nonlinear systems. In the results of [21], [25]–[27], the barrier Lyapunov functions were employed to satisfy the state constraints or guarantee the validity of universal approximation. However, in the proposed scheme of this article, the barrier Lyapunov functions (43) and (44) are utilized to enforce the unknown system dynamics as an unknown bound, which is rejected actively at the last step, thus resulting in a global output feedback control design.

IV. Simulation Results

In this section, we illustrate the above methodology on a ship, which is described by the following model [23]:

$$\ddot{y} + \Phi\dot{y} + b_0(My^3 + Ly) = b_0\, u \tag{65}$$

where y is the course angular velocity, u is the rudder angle, $b_0 \neq 0$, M, and L are unknown constants related to the hydrodynamic coefficients and the mass of the ship. By defining $x_1 = y$, $x_2 = \dot{y} + \Phi y$, we can rewrite the system (65) in the form of (4) as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \varphi(t,y) + \begin{bmatrix}0\\ b_0\end{bmatrix} u \tag{66}$$

where $\varphi(t,y) = [-\Phi y, -b_0(My^3 + Ly)]^T$.

For simulation purpose, it is assumed that $\Phi = 0.2$, $b_0 = 1.85$, $M = 0.12$, and $L = 0.28$. The control objective is to design an adaptive output feedback controller u such that the output y tracks the desired trajectory $y_r = \sin(t)\mathrm{rad/s}$ with steady state error no more than $0.1\mathrm{rad/s}$, minimum speed of convergence as obtained by the exponential e^{-t}.

In the simulation, a funnel function μ is introduced to incorporate the aforementioned transient performance, which is defined as $\mu = \frac{t}{t+1}\frac{1}{(6-0.1)e^{-t}+0.1}$ with $\mu(0) = 0$. Moreover, we choose $k_1 = 2$ and $k_2 = 3$ for the K-filters. The control parameters are given as $c_1 = 20, c_2 = 10, d_2 = 0.5, \gamma_2 = 0.5, \gamma_\varrho = 0.5, \sigma_\varrho = 0.1, \sigma_\theta = 0.3$, and $\Gamma = 0.2$. The initial conditions of (65) are $y(0) = 0.5\mathrm{rad/s}$, $\dot{y}(0) = 0.4\mathrm{rad/s}$, and all other initial values are zero. The simulation results are presented in Figs. 1–4. Apparently, from Figs. 1 and 2, we can see that the output tracking with desired performance specification is achieved, despite the presence of unknown system nonlinearity. The control input u is plotted in Fig. 3. Fig. 4 indicates that the parameter estimation $\hat{\Theta}$ and $\hat{\varrho}$ are bounded. In addition, Fig. 5 shows the tracking error trajectories with various initial conditions, which verifies that for any initial conditions, the prescribed tracking performance can be achieved under the proposed adaptive controller.

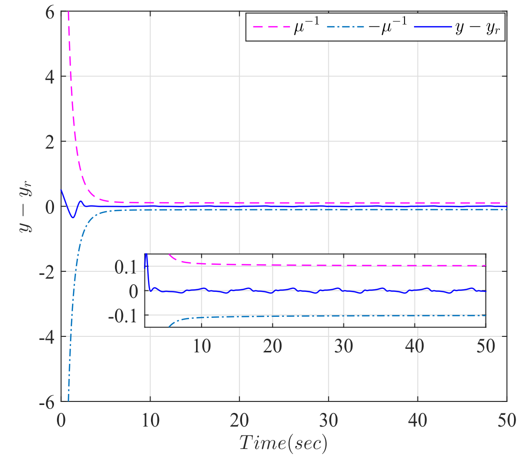

Fig. 2. Output tracking error $y - y_r$ (rad/s).

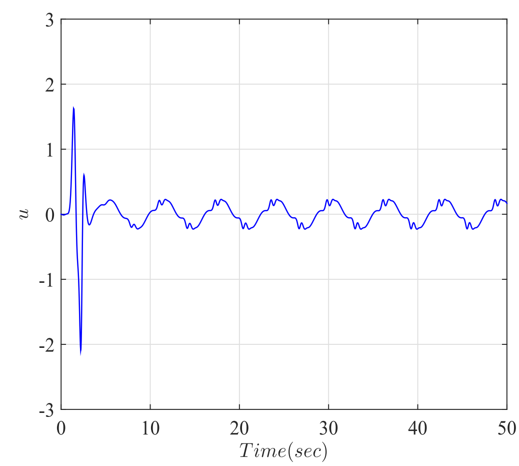

Fig. 3. Control signal u.

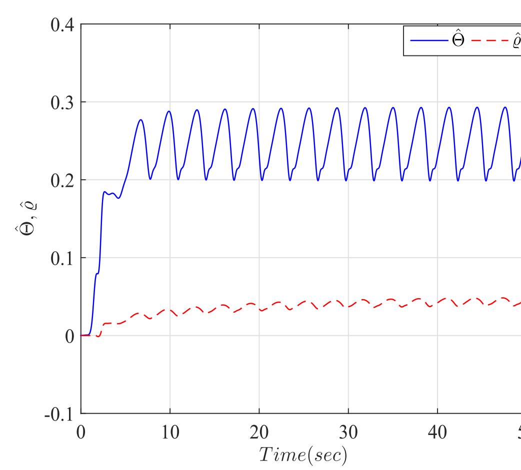

Fig. 4. $\hat{\Theta}$ and $\hat{\varrho}$.

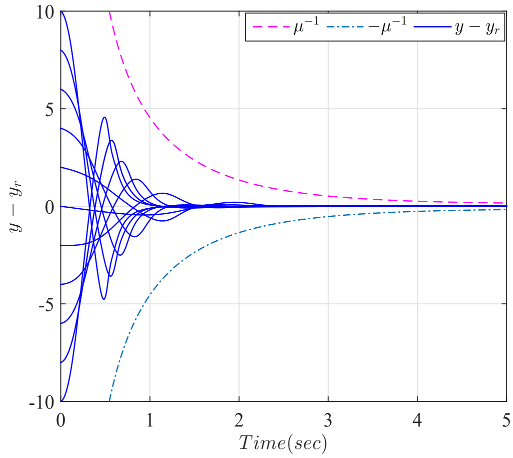

Fig. 5. Output tracking error $y - y_r$ (rad/s) with various initial conditions.

V. Conclusion

In this article, an adaptive output feedback funnel controller, which achieves the output tracking with prescribed transient behavior, is developed for a class of nonlinear systems with arbitrary known relative degree. It is shown that all signals in the closed-loop systems are globally bounded and the output tracking error satisfies the prescribed transient and steady-state performance. Simulation results confirm the effectiveness and the benefits of the theoretical findings. As a future work, we will investigate how to extend the proposed method for nonlinear systems involving unknown nonlinearities coupled with the output and the unmeasured states.

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