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ORIGINAL RESEARCH

Neural networks adaptive control for fractional-order non-linear system with unmodelled dynamics and actuator faults

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Abstract

The fault-tolerant control (FTC) problem for fractional-order (FO) non-linear systems with unmodelled dynamics and actuator faults is studied. First, the author uses neural networks (NNs) to identify unknown non-linear functions and apply a FO dynamic signal to control unmodelled dynamics. Then, fractional-order dynamic surface control (FODSC) is introduced in the design process of the adaptive backstepping control algorithm to avoid complex explosion problems. In addition, an adaptive NNs FTC algorithm using the FO Lyapunov stability criterion is designed. Importantly, the author shows that the proposed system is stable, and the tracking error could be converged to a small neighbourhood of zero. Finally, a simulation example is used to verify the effectiveness of the proposed control scheme.

1 | INTRODUCTION

In recent years, fractional-order non-linear systems (FONSs) have attracted the attention of all circles of society. On one hand, FONSs are used in industrial production to build models, such as hyperchaotic economic systems [1], micro-electro-mechanical resonators [2], and lithium-ion batteries [3]. On the other hand, FONSs are theoretically a natural extension of integer-order non-linear systems (IONSs), and many achievements have been made [4–6]. In [4], a smooth adaptive backstepping control scheme is proposed to ensure the global asymptotic stability of the system. In [5], the author proposes an adaptive tracking controller based on regression. The author further studies this method in [6] in FONS systems with undetectable states. It is worth noting that the controlled object in the above literature considers the case where non-linear functions are known.

It is well known that there are two methods to approach unknown non-linear functions, which are fuzzy logic systems (FLSs) and neural networks (NNs). In literature [7–16], the author uses them to identify unknown non-linear functions in FONS. Among them, the author studied several adaptive intelligent (fuzzy and NN) FONS control algorithms in [7–11]. Furthermore, several intelligent adaptive control strategies using a fractional-order dynamic surface filter (FODSF) have been put forward in [12–14], which do not need to repeatedly derive the control functions, to avoid the problem of 'explosion of complexity'. The authors in [15, 16] put forward two adaptive intelligent decentralized control algorithms for large-scale FONS. One point should be noted that the FONSs in [7–16] are limited to strict-feedback form. Thus, the above-mentioned schemes cannot be applied to solve the control design issues for FONSs in a non-strict-feedback form.

We know that non-strict feedback FONSs have more general research significance than strict feedback FONSs. Because all the variables in the system exist in every non-linear function in non-strict feedback FONS, direct application of the control design method in [7–16] to non-strict feedback FONS will result in algebraic loop problems, which is not allowed [17–20]. Consequently, the authors in [19] proposed a fuzzy-based adaptive control scheme and a fuzzy adaptive output feedback control method for the non-strict-feedback uncertain FONSs, respectively. Afterwards, the authors in [20] put forward an event-triggered NNs adaptive control strategy for the FONSs with unmodelled dynamics and input saturation. However, the above-mentioned works of literature ignore the issue of actuator faults.

In practical engineering, due to the aging and failure of components, actuators or sensors will inevitably fail during the high-load operation. For the actual system, if the system cannot be maintained in time and effectively, the system performance may be degraded or even unstable, resulting in losses. Because of this, the concept of fault-tolerant control (FTC) came into being. FTC is to ensure that the system can still operate stably and meet certain performance indexes in case of sensor, actuator, or component failure. The emergence of FTC has attracted the attention of scholars and has made great progress in decades. Recently, the authors in [21] put forward a robust FTC for fractional-order systems. In [22], an adaptive sliding mode synchronization scheme is proposed for FONSs. The authors in [23] studied the tracking control of triangular FONSs with actuator faults by means of sliding mode control and composite learning sliding mode control. Then, in [8], the author carried out strict feedback adaptive neural network backstepping control for FONSs of actuator failure. However, the control objects of the above works do not take into account the existence of unmodeled dynamics. Therefore, it is of great significance to study FONSs with a non-strict feedback form under the dual condition of unmodelled dynamics and actuator faults.

In this work, motivated by the above-mentioned literature, the NNs adaptive FTC design issue for the FONS in non-strict-feedback form with unmodelled dynamics and actuator faults is presented. The contributions of this work are summarized as follows.

- (i) In the control design, by utilizing the property of NN basis functions and constructing the fractional-order adaptation laws, the issue of an algebraic loop is solved. The existing results in [7–16] also study the control issues of FONSs, but they are all limited to a strict-feedback form.
- (ii) Unlike [20], this paper considers the issues of unmodelled dynamics and actuator faults simultaneously. By introducing an FO dynamic signal and a bound estimation method, the proposed FTC method can effectively compensate for the actuator faults and dominate unmodelled dynamics.
- (iii) The problem of ‘explosion of complexity’ is avoided by applying the FODSC technique in the design of the control strategy.

The remainder of this paper is arranged as follows. In Section 2, I first formulate the FONS system, and give some assumptions and useful lemmas. The main results are stated in Section 3. In Section 4, numerical simulations for validating the results derived are provided. Section 5 concludes this paper.

2 | PROBLEM FORMULATIONS AND PRELIMINARIES

2.1 | System descriptions

The FONS is as follows:

$$
\begin{aligned}
{}_0^C D_t^\alpha z &= q(z, X) \\
{}_0^C D_t^\alpha x_i &= f_i(X) + x_{i+1} + H_i(z, X) \\
{}_0^C D_t^\alpha x_m &= f_m(X) + \sum_{j=1}^{n} u_j + H_m(z, X), \\
y &= x_1
\end{aligned} \tag{1}
$$

where $\alpha \in (0, 1)$, $X_i = [x_1, x_2, \dots, x_i]^T \in \mathbb{R}^i$ $(i = 1, 2, \dots, m, X = X_m)$, is the system state vector. $f_i(\cdot)$ represent smooth unknown non-linear functions. z represents unmodelled dynamics. The dynamical disturbances are represented by $H_i(z, X)$. $q(\cdot)$ and $H_i(\cdot)$ are uncertain functions. y denotes the system output. Besides,$u = [u_1, u_2, \dots, u_n]^T \in \mathbb{R}^n$ represents the input vector of components that may fail during system operation.

Similar to [9], the actuator faults can be modelled as follows:

$$
\begin{aligned}
u_j^F(t) &= \mu_{j,b} u_j(t) + \bar{u}_{j,b}(t), \quad t \in [t_{j,b}^s, t_{j,b}^e] \\
\mu_{j,b} \bar{u}_j(t) &= 0, \quad j = 1, \dots, m, b = 1, 2, \dots
\end{aligned} \tag{2}
$$

where $\mu_{j,b} \in [0, 1]$, $t_{j,b}^s$ and $t_{j,b}^e$ represent the start and end times of fault occurrence. $\bar{u}_{j,b}(t)$ is an unknown constant.

1. $\mu_{j,b} = 1$ and $\bar{u}_{j,b}(t) = 0$. There are no faults in actuators.
2. $0 < \underline{\mu}_{j,b} \leq \mu_{j,b} \leq \bar{\mu}_{j,b} < 1$ and $\bar{u}_{j,b} = 0$. This case means the partial loss of effectiveness.
3. $\mu_{j,b} = 0$ and $\bar{u}_{j,b}(t) \neq 0$. It implies total loss of effectiveness.

Assumption 1 [8, 24, 25]. *When any $n-1$ actuator fails such as (2), other actuators may strike, but the closed-loop system can still be driven to achieve the control objective, which is the construction principle of system (1).*

Assumption 2 [9]. *There exist unknown positive constants $\bar{\bar{u}}_{j,b}$, such that $|\bar{u}_{j,b}(t)| \leq \bar{\bar{u}}_{j,b}$.*

Assumption 3 [11, 12]. *y_d are the given reference signals, and they are sufficiently smooth functions of t and y_d, ${}_0^C D_t^\alpha y_d$ and ${}_0^C D_t^\alpha({}_0^C D_t^\alpha y_d)$ are bounded. In addition, suppose there is a constant $B > 0$ such that*

$$
y_d^2 + ({}_0^C D_t^\alpha y_d)^2 + ({}_0^C D_t^\alpha({}_0^C D_t^\alpha y_d))^2 \leq B. \tag{3}
$$

Assumption 4 [20]. *The system ${}_0^C D_t^\alpha z = q(z, X)$ has a Mittag-Leffler ISpS Lyapunov function $V_z(z)$ such that*

$$
\alpha_1(|z|) \leq V_z(z) \leq \alpha_2(|z|), \tag{4}
$$

$$
{}_0^C D_t^\alpha V_z(z) \leq -c V_z(z) + \gamma(|y|) + d, \tag{5}
$$

where $c > 0$ and $d > 0$ are known constants and α_1, α_2, γ are k_∞ functions.

Assumption 5 [24, 26, 27]. *There exists an unknown positive constant δ_i^*, such that*

$$
H_i(z, X) \leq \delta_i^* \chi_{i,1}(y) + \delta_i^* \chi_{i,2}(|z|), \tag{6}
$$

where $\chi_{i,1}(\cdot)$ and $\chi_{i,2}(\cdot)$ are two known nonnegative smooth functions, $\chi_{i,2}(0) = 0$.

2.1.1 | Control objectives

In this work, the objective is to design a stable NNs adaptive FTC algorithm for plant (1) with unmodelled dynamics and actuator faults. And it satisfies two conditions, one is all the signals in the closed-loop system are bounded, the other is the output y_i could track the given reference signal y_d as closely as possible.

2.2 | Preliminaries

Definition 1 [28]: Define the αth Caputo fractional derivative of the following form:

$$ {}_0^C D_t^\alpha F(t) = \frac{1}{\Gamma(\omega-\alpha)} \int_0^t \frac{F^{(\omega)}(\tau)}{(t-\tau)^{\alpha+1-\omega}} d\tau, \tag{7}$$

where $\omega - 1 \le \alpha \le \omega, \omega \in N^+$. $\Gamma(\cdot) = \int_0^{+\infty} \tau^{\cdot-1} e^{-\tau} d\tau$ represent the Euler Gamma function, and $\Gamma(1) = 1$.

Definition 2 [28]: Define the Mittag–Leffler function of the following form:

$$ E_{\alpha,\phi}(\gamma) = \sum_{j=0}^{\infty} \frac{\gamma^j}{\Gamma(j\alpha + \phi)}, \tag{8}$$

where $\alpha, \phi \in \mathbb{R}^+$ and $\gamma \in \mathbb{C}$. Its Laplace transform is

$$ L\{t^{\phi-1} E_{\alpha,\phi}(-\kappa t^\alpha)\} = \frac{s^{\alpha-\phi}}{s^\alpha + \kappa}, \tag{9}$$

where $\kappa \in \mathbb{R}$.

Lemma 1 [29]. *Let α satisfy $\alpha \in (0,2)$, $\beta \in \mathbb{R}$and $\delta \in (\pi\alpha/2, \min\{\pi, \pi\alpha\})$, then one has*

$$ E_{\alpha,\beta}(\zeta) \le \frac{\lambda}{1 + |\zeta|}, \tag{10}$$

where $\lambda > 0, |\zeta| \ge 0$ and $\delta \le |\arg(\zeta)| \le \pi$.

Lemma 2 [20]. *For any constants $\bar{c}$ in $(0,c)$, any initial condition $z_0 = z(t_0)$ and $r_0 > 0$, we suppose system (1) has a Mittag–Leffler ISpS Lyapunov function, then for a function $\bar{\gamma}(y) \ge \gamma(|y|)$, there exists a finite $T^0 = T^0(c, r, z) \ge 0$, a non-negative function $D(t_0, t)$ defined for all $t \ge t_0$ and a signal described by*

$$ {}_0^C D_t^\alpha r = -\bar{c} r + \bar{\gamma}(y) + \bar{d}, \quad r(t_0) = r_0, \tag{11}$$

such that $D(t_0, t) = 0$ for all $t \ge t_0 + T^0$ and

$$ V_z \le r + D(t_0, t), \tag{12}$$

for all $t \ge t_0$.

Radial basis function neural networks (RBFNNs) [8, 10, 30, 31] can be described as

$$ h_{nn}(X) = w^\tau \psi(X) \tag{13}$$

In (13), the input vector $X \in \Omega \subset \mathbb{R}^q$, the vector $w = [w_1, \dots, w_s]^\tau$, the NN node number $s > 1$; $\psi(X) = [\psi_1(X), \dots, \psi_s(X)]^\tau$, and $\psi_i(X)$ are selected as the commonly utilized Gaussian function, its form is as follows:

$$ \psi_i(X) = \exp\left[\frac{-(X-\eta_i)^\tau (X-\eta_i)}{\mu_i^2}\right], i = 1, \dots, s $$

where $\mu_i = [\mu_{i,1}, \dots, \mu_{i,q}]^\tau$ represents the centre of the receptive field and μ_i expresses the width of the Gaussian function.

Based on [8, 10, 31], RBFNNs can be used for any continuous function $h(X)$ as

$$ h(X) = w^{*\tau} \psi(X) + \tau(X) \tag{14}$$

where w_i^* represents the ideal parameter vectors and $\tau(X)$ represents the approximation error, which satisfies $|\tau_i| \le \tau_i^*$ with $\tau_i^* > 0$.

Remark 1. Note that RBFNNs are introduced to solve unknown non-linear functions in the controlled systems (1) because they have the property of approximating unknown non-linear functions in compact sets. Moreover, there are some other non-linear approximators, such as FLSs [19, 32], which can replace RBFNNs and achieve the same results.

3 | NNS ADAPTIVE CONTROL DESIGN AND STABILITY ANALYSIS

Consider the coordinate transformations which has the following form:

$$ \begin{aligned} \zeta_1 &= x_1 - y_d \\ \zeta_i &= x_i - \kappa_{i-1} \quad i = 2, \dots, m \\ \nu_{i-1} &= \kappa_{i-1} - \varpi_{i-1} \end{aligned} \tag{15}$$

where the tracking error is represented by ζ_1, while the dynamic surface errors are represented by ζ_i. The FODSF variables are represented by $\kappa_i$and the FODSF output errors are represented by ν_i. $W_i^* = ||w_i^*||^2$, W_i and υ_i are the estimations of w_i^* and υ_i^*, where $\tilde{w}_i = w_i^* - w_i$ and $\tilde{\upsilon}_i = \upsilon_i^* - \upsilon_i$.

Step$i$, 1: Using (1) and (15),

$$ \begin{aligned} {}_0^C D_t^\alpha \zeta_1 &= {}_0^C D_t^\alpha x_1 - {}_0^C D_t^\alpha y_d \\ &= \zeta_2 + \varpi_1 + \nu_1 + H_1(z, X) + \tau_1 \\ &\quad + w_1^{*\tau} \psi_1(X) - {}_0^C D_t^\alpha y_d \end{aligned} \tag{16}$$

Here, we choose the Lyapunov function candidate which has the following form:

$$V_1 = \frac{1}{2}\zeta_1^2 + \frac{1}{2p_1}\tilde{W}_1^2 + \frac{1}{2\bar{p}_1}\tilde{v}_1^2 + \frac{r}{\underline{p}} \tag{17}$$

where $p_1 > 0, \bar{p}_1 > 0$ and $\underline{p} > 0$ are design constants.

Since ${}_0^C D_t^\alpha(x^\tau(t)x(t))/2 \le x^\tau(t){}_0^C D_t^\alpha x(t)$ in [33], using (16) and (17), thus

$$\begin{aligned} {}_0^C D_t^\alpha V_1 \le \zeta_1(&\zeta_2 + \varpi_1 + \nu_1 + \tau_1 + H_1(z, X) \\ &+ w_1^{*\tau}\psi_1(X) - {}_0^C D_t^\alpha y_d) + \frac{1}{\underline{p}}{}_0^C D_t^\alpha r \\ &- \frac{1}{p_1}\tilde{w}_1^\tau {}_0^C D_t^\alpha w_1 - \frac{1}{\bar{p}_1}\tilde{v}_1 {}_0^C D_t^\alpha v_1 \end{aligned} \tag{18}$$

By the Young inequality, the property of $0 < \psi_1^\tau(\cdot)\psi_1(\cdot) \le s$ and Assumption 5, we can obtain

$$\zeta_1(\tau_1 + \nu_1) \le \zeta_1^2 + \frac{\nu_1^2}{2} + \frac{\tau_1^{*2}}{2} \tag{19}$$

$$\zeta_1 w_1^{*\tau}\psi_1(X) \le \frac{\zeta_1^2 W_1^*}{4} + s \tag{20}$$

$$\zeta_1 H_1(z, X) \le |\zeta_{i,1}|\delta_1^* \chi_{1,1}(X_1) + |\zeta_{i,1}|\delta_1^*\chi_{1,2}(|z|) \tag{21}$$

$$\begin{aligned} |\zeta_1|\delta_1^*\chi_{1,2}(|z|) &\le |\zeta_1|\delta_1^*\chi_{1,2}\circ\alpha_1^{-1}(2r(t)) \\ &\quad + |\zeta_1|\delta_1^*\chi_{1,2}\circ\alpha_1^{-1}(2D(t_0 + T)) \\ &\le |\zeta_1|\delta_1^*\chi_{1,2}\circ\alpha_1^{-1}(2r(t)) \\ &\quad + \frac{1}{2}\zeta_1^2\delta_1^{*2} + d_1(t_0, t) \end{aligned} \tag{22}$$

where $d_1(t_0, t) = \frac{1}{2}(\chi_{1,2}\circ\alpha_1^{-1}(2D(t_0 + T)))^2$.

Then, by using the inequality $0 \le |a| - a\tanh(\frac{a}{\varepsilon}) < 0.2785q'$ in [30], one can obtain

$$|\zeta_1|\delta_1^*\chi_{1,1}(X_1) \le \zeta_1\delta_1^*\lambda_{1,1} + \delta_1^* 0.2785q_1 \tag{23}$$

$$|\zeta_1|\delta_1^*\chi_{1,2}\circ\alpha_1^{-1}(2r(t)) \le \zeta_1\delta_1^*\lambda_{1,2} + \delta_1^* 0.2785q_2 \tag{24}$$

where $q_1 > 0$, $q_2 > 0$, $\lambda_{1,1} = \chi_{1,1}(X_1)\tanh(\frac{\zeta_1\chi_{1,1}(X_1)}{q_1})$ and $\lambda_{1,2} = \chi_{1,2}\circ\alpha_1^{-1}(2r(t))\tanh(\frac{\zeta_1\chi_{1,2}\circ\alpha_1^{-1}(2r(t))}{q_2})$.

With the consideration of (21)–(24), one can obtain

$$\zeta_1 H_1(z, X) \le \zeta_1 v_1^*\lambda_1 + v_1^* q + \frac{1}{2}\zeta_1^2 v_1^* + d_1(t_0, t) \tag{25}$$

where $v_1^* = \max\{1, \delta_1^*, \delta_1^{*2}\}$, $\lambda_1 = \lambda_{1,1} + \lambda_{1,2}$ and $q = 0.2785q_1 + 0.2785q_2$.

Substituting (19), (20) and (25) into (18) results in

$$\begin{aligned} {}_0^C D_t^\alpha V_1 \le \zeta_1\Big(&\zeta_2 + \varpi_1 + \frac{1}{2}\zeta_1 v_1^* + \zeta_1 + v_1^*\lambda_1 + \frac{1}{4}W_1^*\zeta_1 \\ &- {}_0^C D_t^\alpha y_d\Big) - \frac{1}{p_1}\tilde{W}_1 {}_0^C D_t^\alpha W_1 - \frac{1}{\bar{p}_1}\tilde{v}_1 {}_0^C D_t^\alpha v_1 \\ &+ v_1^* q + d_1(t_0, t) + \frac{1}{\underline{p}}(-\bar{c}r + \bar{\gamma}(x_1) + \bar{d} \\ &+ \zeta_1^2\bar{\gamma}(x_1) - \zeta_1^2\bar{\gamma}(x_1)) + \frac{\nu_1^2}{2} + \frac{\tau_1^{*2}}{2} + s \end{aligned} \tag{26}$$

Design the virtual controller ϖ_1, the adaptation laws ${}_0^C D_t^\alpha W_1$ and ${}_0^C D_t^\alpha v_1$ as

$$\begin{aligned} \varpi_1 = &-c_1\zeta_1 - \frac{1}{4}W_1\zeta_1 - \frac{1}{2}\zeta_1 v_1 - \zeta_1 \\ &- \underline{p}^{-1}\zeta_1\bar{\gamma}(x_1) - v_1\lambda_1 + {}_0^C D_t^\alpha y_d \end{aligned} \tag{27}$$

$${}_0^C D_t^\alpha W_1 = \frac{1}{4}\zeta_1^2 p_1 - \beta_1 W_1 \tag{28}$$

$${}_0^C D_t^\alpha v_1 = \bar{p}_1\zeta_1\lambda_1 + \frac{1}{2}\bar{p}_1\zeta_1^2 - \bar{\beta}_1 v_1 \tag{29}$$

where $\beta_1 > 0$ and $\bar{\beta}_1 > 0$ are design parameters.

In [20, 34], $\exists\lambda > 0$ that can make the inequality $\frac{1}{\underline{p}}(1 - \zeta_1^2)\bar{\gamma}(x_1) \le \lambda$ hold. Thus, substituting (27)–(29) into (26), we have

$$\begin{aligned} {}_0^C D_t^\alpha V_1 \le \zeta_2\zeta_1 - c_1\zeta_1^2 + \frac{\beta_1}{p_1}\tilde{W}_1 W_1 + \rho_1 \\ + \frac{\bar{\beta}_1}{\bar{p}_1}\tilde{v}_1 v_1 - \frac{\bar{c}}{\underline{p}}r + \frac{\nu_1^2}{2} \end{aligned} \tag{30}$$

where $\rho_1 = v_1^* q + \frac{\tau_1^{*2}}{2} + d_1(t_0, t) + \lambda + \frac{\bar{d}}{\underline{p}} + s$.

Remark 2. By adopting the property of NN basis functions $0 < \psi_i(\cdot)\psi_i(\cdot) \le s$ and introducing a fractional-order parameter adaptation law ${}_0^C D_t^\alpha W_1$, it can be seen from (27) that the virtual controller and fractional-order adaptation laws only contain a partial variable x_1, not the variables x, thus avoiding the generation of algebraic loop issue. It should be noted that different from references [7–16], if the above-mentioned results are directly applied to the controlled systems (1), the issue of an algebraic loop will arise, which is not allowed.

Introduce a FODSF in [12] as

$$\sigma_1 {}_0^C D_t^\alpha \kappa_1 + \kappa_1 = \varpi_1, \quad \kappa_1(0) = \varpi_1(0) \tag{31}$$

where σ_1 is a constant.

By using (15) and (31), one has

$$
\begin{aligned}
{}_0^C D_t^\alpha \nu_1 &= {}_0^C D_t^\alpha \kappa_1 - {}_0^C D_t^\alpha \varpi_1 \\
&= -\frac{\nu_1}{\sigma_1} + D_1(.)
\end{aligned} \tag{32}
$$

where $D_1(\cdot)$ is a continuous function.

Step i: Using (1) and (15), we have

$$
\begin{aligned}
{}_0^C D_t^\alpha \zeta_i &= {}_0^C D_t^\alpha x_i - {}_0^C D_t^\alpha \kappa_{i-1} \\
&= \zeta_{i+1} + \nu_i + \varpi_i + w_i^{*\tau}\psi_i(X) + \tau_i \\
&\quad + H_i(z, X) - {}_0^C D_t^\alpha \kappa_{i-1}
\end{aligned} \tag{33}
$$

Then we choose the Lyapunov function candidate as

$$
V_i = V_{i-1} + \frac{1}{2}\zeta_i^2 + \frac{1}{2p_i}\tilde{W}_i^2 + \frac{1}{2\bar{p}_i}\tilde{\upsilon}_i^2 + \frac{1}{2}\nu_{i-1}^2 \tag{34}
$$

where $p_i > 0$ and $\bar{p}_i > 0$ are design parameters.

Similarly, using (33) and (35), and the inequality in [33], we have

$$
\begin{aligned}
{}_0^C D_t^\alpha V_i \le{}& {}_0^C D_t^\alpha V_{i-1} + \zeta_i\big(\zeta_{i+1} + \nu_i + \varpi_i + w_i^{*\tau}\psi_i(X) \\
&+ \tau_i + H_i(z, X) - {}_0^C D_t^\alpha \kappa_{i-1}\big) - \frac{1}{p_i}\tilde{W}_{i0}^C D_t^\alpha W_i \\
&- \frac{1}{\bar{p}_i}\tilde{\upsilon}_{i0}^C D_t^\alpha \upsilon_i + \nu_{i-1}{}_0^C D_t^\alpha \nu_{i-1}
\end{aligned} \tag{35}
$$

By the Young inequality, the property of $0 < \psi_1^\tau(\cdot)\psi_1(\cdot) \le s$ and Assumption 5, the following results hold:

$$
\zeta_i(\nu_i + \tau_i) \le \zeta_i^2 + \frac{\tau_i^{*2}}{2} + \frac{\nu_i^2}{2} \tag{36}
$$

$$
\zeta_i w_i^{*\tau}\psi_i(X) \le \frac{\zeta_i^2 W_i^*}{4} + s \tag{37}
$$

$$
\zeta_i H_i(z_i, X) \le |\zeta_i|\delta_i^*(\chi_{i,1}(|X_1|) + \chi_{i,2}(|z|)) \tag{38}
$$

$$
\begin{aligned}
|\zeta_i|\delta_i^*\chi_{i,2}(|z|) \le{}& |\zeta_i|\delta_i^*\chi_{i,2}\circ\alpha_1^{-1}(2r(t)) \\
&+ |\zeta_i|\delta_i^*\chi_{i,2}\circ\alpha_1^{-1}(2D(t_0 + T)) \\
\le{}& |\zeta_i|\delta_i^*\chi_{i,2}\circ\alpha_1^{-1}(2r(t)) + \frac{1}{2}\delta_i^{*2}\zeta_i^2 + d_i(t_0, t)
\end{aligned} \tag{39}
$$

where $d_i(t_0, t) = \frac{1}{2}(\chi_{i,2}\circ\alpha_1^{-1}(2D(t_0 + T)))^2$.

Then, by the inequality $0 \le |a| - a\tanh(\frac{a}{\varepsilon}) < 0.2785q'$ in [35], one has

$$
|\zeta_i|\delta_i^*\chi_{i,1}(|X_i|) \le \zeta_i\delta_i^*\lambda_{i,1} + 0.2785\delta_i^* q_1 \tag{40}
$$

$$
|\zeta_i|\delta_i^*\chi_{i,2}\circ\alpha_1^{-1}(2r(t)) \le \zeta_i\delta_i^*\lambda_{i,2} + 0.2785\delta_i^* q_2 \tag{41}
$$

where $\lambda_{i,1} = \chi_{i,1}(|X_1|)\tanh(\frac{\zeta_i\chi_{i,1}(|X_1|)}{q_1})$ and $\lambda_{i,2} = \chi_{i,2}\circ\alpha_1^{-1}(2r(t))\tanh(\frac{\zeta_i\chi_{i,2}\circ\alpha_1^{-1}(2r(t))}{q_2})$.

On the basis of (38)–(41), the following inequality holds:

$$
\zeta_i H_i(z_i, X) \le \zeta_i\upsilon_i^*\lambda_i + q\upsilon_i^* + \frac{1}{2}\upsilon_i^*\zeta_i^2 + d_i(t_0, t) \tag{42}
$$

where $\lambda_i = \lambda_{i,1} + \lambda_{i,2}$ and $\upsilon_i^* = \max\{1, \delta_i^*, \delta_i^{*2}\}$.

Substituting (36), (37) and (42) into (35) results in

$$
\begin{aligned}
{}_0^C D_t^\alpha V_i \le{}& {}_0^C D_t^\alpha V_{i-1} + \zeta_i(\zeta_{i+1} + \varpi_i + \zeta_i + \upsilon_i^*\lambda_i - {}_0^C D_t^\alpha \kappa_{i-1} \\
&+ \frac{\upsilon_i^*\zeta_i}{2} + \frac{1}{4}\zeta_i W_i^*) + q\upsilon_i^* + d_i(t_0, t) + \frac{\tau_i^{*2}}{2} + \frac{\nu_i^2}{2} \\
&- \frac{1}{p_i}\tilde{W}_{i0}^C D_t^\alpha W_i - \frac{1}{\bar{p}_i}\tilde{\upsilon}_{i0}^C D_t^\alpha \upsilon_i + \nu_{i-1}{}_0^C D_t^\alpha \nu_{i-1} + s
\end{aligned} \tag{43}
$$

Here, we design the virtual controller ϖ_i and the adaptation laws ${}_0^C D_t^\alpha W_i$ and ${}_0^C D_t^\alpha \upsilon_i$ as

$$
\varpi_i = -c_i\zeta_i - \frac{1}{4}W_i\zeta_i - \zeta_{i-1} - \zeta_i - \frac{1}{2}\upsilon_i\zeta_i - \upsilon_i\lambda_i + {}_0^C D_t^\alpha \kappa_{i-1} \tag{44}
$$

$$
{}_0^C D_t^\alpha W_i = \frac{1}{4}p_i\zeta_i^2 - \beta_i W_i \tag{45}
$$

$$
{}_0^C D_t^\alpha \upsilon_i = \bar{p}_i\zeta_i\lambda_i + \frac{1}{2}\bar{p}_i\zeta_i^2 - \bar{\beta}_i\upsilon_i \tag{46}
$$

where $c_i > 0$, $\beta_i > 0$ and $\bar{\beta}_i > 0$ are design parameters.

Substituting (44)–(46) into (43), we get

$$
\begin{aligned}
{}_0^C D_t^\alpha V_i \le{}& \sum_{h=1}^{i}\left(-c_h\zeta_h^2 + \frac{\beta_h}{p_h}\tilde{W}_h W_h + \frac{\bar{\beta}_h}{\bar{p}_h}\tilde{\upsilon}_h\upsilon_h + \frac{\nu_h^2}{2}\right) \\
&+ \sum_{h=1}^{i-1}\left(\nu_h\left(-\frac{\nu_h}{\sigma_h} + D_h(.)\right)\right) - \frac{\bar{c}}{\underline{p}}r + \zeta_i\zeta_{i+1} + \rho_i
\end{aligned} \tag{47}
$$

where $\rho_i = \rho_{i-1} + q\upsilon_i^* + \frac{\tau_i^{*2}}{2} + d_i(t_0, t) + s$.

Suppose the virtual controller ϖ_i passes through a FODSF with a constant σ_i; therefore, $\kappa_i$satisfies

$$\sigma_i {}_0^C D_t^\alpha \kappa_i + \kappa_i = \varpi_i, \quad \kappa_i(0) = \varpi_i(0) \tag{48}$$

By using (15) and (48), one can obtain

$$\begin{aligned} {}_0^C D_t^\alpha \nu_i &= {}_0^C D_t^\alpha \kappa_i - {}_0^C D_t^\alpha \varpi_i \\ &= -\frac{\nu_i}{\sigma_i} + D_i(.) \end{aligned} \tag{49}$$

where $D_i(\cdot)$ is a continuous function.

Step m: Using (1) and (15), we get

$$\begin{aligned} {}_0^C D_t^\alpha \zeta_m &= {}_0^C D_t^\alpha x_m - {}_0^C D_t^\alpha \kappa_{m-1} \\ &= w_m^{*\tau} \psi_m(X) + \tau_m + \sum_{j=1}^n (\mu_j u_j + \bar{u}_j) + H_m(z, X) \\ &\quad - {}_0^C D_t^\alpha \kappa_{m-1} \end{aligned} \tag{50}$$

From Assumptions 1–2, we know that $\sum_{j=1}^n \mu_j \geq \min\{\underline{\mu}_1, \underline{\mu}_2 \dots \underline{\mu}_n\} > 0$ for all $t > 0$. Thus $\inf_{t \geq 0} \sum_{j=1}^n \mu_j \geq \min\{\underline{\mu}_1, \underline{\mu}_2, \dots, \underline{\mu}_n\} > 0$. Define

$$\theta^* = \inf_{t \geq 0} \sum_{j=1}^n \mu_j, \quad \Theta^* = \frac{1}{\theta^*}, \quad \varepsilon^* = \sup_{t \geq 0} \sum_{j=1}^n \bar{u}_j \tag{51}$$

where Θ and ε are the estimations of Θ^* and ε^*. $\tilde{\Theta} = \Theta^* - \Theta$ and $\tilde{\varepsilon} = \varepsilon^* - \varepsilon$.

We chose the whole Lyapunov function candidate as

$$\begin{aligned} V &= V_{m-1} + \frac{1}{2}\zeta_m^2 + \frac{1}{2p_m}\tilde{W}_m^2 + \frac{1}{2\bar{p}_m}\tilde{\upsilon}_m^2 \\ &\quad + \frac{1}{2}\nu_{m-1}^2 + \frac{\theta^*}{2p_{11}}\tilde{\Theta}^2 + \frac{1}{2p_{22}}\tilde{\varepsilon}^2 \end{aligned} \tag{52}$$

where $p_m > 0$, $\bar{p}_m > 0$, $p_{11} > 0$ and $p_{22} > 0$ are design parameters.

Similarly, from (50) and (52), and by utilizing the inequality in [33], we get

$$\begin{aligned} {}_0^C D_t^\alpha V &\leq {}_0^C D_t^\alpha V_{m-1} + \zeta_m \Bigg(w_m^{*\tau} \psi_m(X_i) + \tau_m + H_m(z, X) \\ &\quad + \sum_{j=1}^n \mu_j u_j + \sum_{j=1}^n \bar{u}_j - {}_0^C D_t^\alpha \kappa_{m-1} \Bigg) - \frac{1}{p_m} \tilde{W}_m {}_0^C D_t^\alpha W_m \\ &\quad - \frac{1}{p_{22}} \tilde{\varepsilon} {}_0^C D_t^\alpha \varepsilon - \frac{1}{\bar{p}_m} \tilde{\upsilon}_m {}_0^C D_t^\alpha \upsilon_m \\ &\quad + \nu_{m-1} {}_0^C D_t^\alpha \nu_{m-1} - \frac{\theta^*}{p_{11}} \tilde{\Theta} {}_0^C D_t^\alpha \Theta \end{aligned} \tag{53}$$

By the Young inequality, the property of $0 < \psi_1^\tau(\cdot)\psi_1(\cdot) \leq s$ and Assumption 5, the following results hold:

$$\zeta_m \tau_m \leq \frac{\tau_m^{*2}}{2} + \frac{\zeta_m^2}{2} \tag{54}$$

$$\zeta_m w_m^{*\tau} \psi_m(X) \leq \frac{\zeta_m^2 W_m^*}{4} + s \tag{55}$$

$$\zeta_m H_m(z, X) \leq \delta_m^* |\zeta_m| (\chi_{m,1}(|X_1|) + \chi_{m,2}(|z|)) \tag{56}$$

$$\begin{aligned} \delta_m^* |\zeta_m| \chi_{m,2}(|z|) &\leq \delta_m^* |\zeta_m| \chi_{m,2} \circ \alpha_1^{-1}(2r(t)) \\ &\quad + \delta_m^* |\zeta_m| \chi_{m,2} \circ \alpha_1^{-1}(2D(t_0 + T)) \\ &\leq \delta_m^* |\zeta_m| \chi_{m,2} \circ \alpha_1^{-1}(2r(t)) \\ &\quad + \frac{1}{2}\zeta_m^2 \delta_m^{*2} + d_m(t_0, t) \end{aligned} \tag{57}$$

where $d_m(t_0, t) = \frac{1}{2}(\chi_{m,2} \circ \alpha_1^{-1}(2D(t_0 + T)))^2$.

Then, by the inequality $0 \leq |a| - a \tanh(\frac{a}{\varepsilon}) < 0.2785q'$ in [35], the following inequality holds:

$$\delta_m^* |\zeta_m| \chi_{m,1}(|X|) \leq \delta_m^* \zeta_m \lambda_{m,1} + 0.2785 \delta_m^* q_1 \tag{58}$$

$$|\zeta_m| \delta_m^* \chi_{m,2} \circ \alpha_1^{-1}(2r(t)) \leq \delta_m^* \zeta_m \lambda_{m,2} + 0.2785 \delta_m^* q_2 \tag{59}$$

where $\lambda_{m,1} = \chi_{m,1}(|X_1|) \tanh(\frac{\zeta_m \chi_{m,1}(|X_1|)}{q_1})$ and $\lambda_{m,2} = \chi_{m,2} \circ \alpha_1^{-1}(2r(t)) \tanh(\frac{\zeta_m \chi_{m,2} \circ \alpha_1^{-1}(2r(t))}{q_2})$.

On the basis of (56)–(59), we get

$$\zeta_m H_m(z, X) \leq \upsilon_m^* \zeta_m \lambda_m + \upsilon_m^* q + \frac{1}{2}\zeta_m^2 \upsilon_m^* + d_m(t_0, t) \tag{60}$$

where $\lambda_m = \lambda_{m,1} + \lambda_{m,2}$ and $\upsilon_m^* = \max\{1, \delta_m^*, \delta_m^{*2}\}$.

Substituting (54), (55) and (60) into (53) results in

$$\begin{aligned} {}_0^C D_t^\alpha V &\leq {}_0^C D_t^\alpha V_{m-1} + \zeta_m \Bigg(\frac{1}{4}\zeta_m W_m^* + \frac{\zeta_m}{2} + H_m(z, X) \\ &\quad + \sum_{j=1}^n \mu_j u_j + \sum_{j=1}^n \bar{u}_j - {}_0^C D_t^\alpha \kappa_{m-1} + \upsilon_m^* \lambda_m \\ &\quad + \frac{1}{2}\zeta_m \upsilon_m^* \Bigg) - \frac{1}{p_m} \tilde{w}_m^\tau {}_0^C D_t^\alpha w_m + \frac{\tau_m^{*2}}{2} + s \\ &\quad - \frac{1}{\bar{p}_m} \tilde{\upsilon}_m {}_0^C D_t^\alpha \upsilon_m + \nu_{m-1} {}_0^C D_t^\alpha \nu_{m-1} + d_m(t_0, t) \\ &\quad + \upsilon_m^* q - \frac{\theta^*}{p_{11}} \tilde{\Theta} {}_0^C D_t^\alpha \Theta - \frac{1}{p_{22}} \tilde{\varepsilon} {}_0^C D_t^\alpha \varepsilon \end{aligned} \tag{61}$$

Design the virtual and actual controllers ϖ_m and u_j as

$$\varpi_m = c_m\zeta_m + \zeta_{m-1} + \frac{\zeta_m}{2} + \frac{1}{4}W_m\zeta_m + \upsilon_m\lambda_m + \frac{1}{2}\zeta_m\upsilon_m + \varepsilon\tanh\left(\frac{\zeta_m}{q_3}\right) - {}_0^C D_t^\alpha \kappa_{m-1} \tag{62}$$

$$u_j = -\frac{\zeta_m\Theta^2\varpi_m^2}{\sqrt{\zeta_m^2\Theta^2\varpi_m^2 + \omega^2}} \tag{63}$$

By adding and subtracting ϖ_m and $\varepsilon^*\zeta_m\tanh(\frac{\zeta_m}{q_3})$ on the right-hand side of (61), the following inequality can be obtained:

$$\begin{aligned} {}_0^C D_t^\alpha V \le {}& {}_0^C D_t^\alpha V_{m-1} + \zeta_m\left(\frac{1}{4}\zeta_m\tilde{W}_m + H_m(z,X) + \sum_{j=1}^n \rho_j u_j + \tilde{\upsilon}_m\lambda_m - c_m\zeta_m + \frac{1}{2}\zeta_m\tilde{\upsilon}_m + \varpi_m\right) + |\zeta_m|\varepsilon^* + \frac{\tau_m^{*2}}{2} \\ & - \frac{1}{p_m}\tilde{w}_m^\tau {}_0^C D_t^\alpha w_m + s - \frac{1}{\bar{p}_m}\tilde{\upsilon}_m {}_0^C D_t^\alpha \upsilon_m + \nu_{m-1} {}_0^C D_t^\alpha \nu_{m-1} \\ & + d_m(t_0,t) + \upsilon_m^* q - \frac{\theta^*}{p_{11}}\tilde{\Theta} {}_0^C D_t^\alpha \Theta - \zeta_m\zeta_{m-1} \\ & - \varepsilon\zeta_m\tanh\left(\frac{\zeta_m}{q_3}\right) - \frac{1}{p_{22}}\tilde{\varepsilon} {}_0^C D_t^\alpha \varepsilon + \varepsilon^*\zeta_m\tanh\left(\frac{\zeta_m}{q_3}\right) \\ & - \varepsilon^*\zeta_m\tanh\left(\frac{\zeta_m}{q_3}\right) \end{aligned} \tag{64}$$

Using the inequalities $0 \le |a| - a\tanh(\frac{a}{\varepsilon}) < 0.2785q'$ and $|z| - \frac{z^2}{\sqrt{z^2+\omega^2}} \le \omega$, we can obtain

$$\zeta_m\sum_{j=1}^n \rho_j u_j \le -\frac{\theta^*\zeta_m^2\Theta^2\varpi_m^2}{\sqrt{\zeta_m^2\Theta^2\varpi_m^2 - \omega^2}} \le \omega\theta^* - \zeta_m\varpi_m\theta^*\Theta \tag{65}$$

$$\zeta_m\varpi_m - \zeta_m\varpi_m\theta^*\Theta = \zeta_m\varpi_m\theta^*\Theta^* - \zeta_m\varpi_m\theta^*\Theta = \zeta_m\varpi_m\theta^*\tilde{\Theta} \tag{66}$$

$$|\zeta_m|\varepsilon^* - \varepsilon^*\zeta_m\tanh\left(\frac{\zeta_m}{q_3}\right) \le 0.2785\varepsilon^* q_3 \tag{67}$$

Design the adaptation laws ${}_0^C D_t^\alpha W_m$, ${}_0^C D_t^\alpha \upsilon_m$, ${}_0^C D_t^\alpha \Theta$ and ${}_0^C D_t^\alpha \varepsilon$ as

$${}_0^C D_t^\alpha W_m = \frac{1}{4}\zeta_m^2 p_m - \beta_m W_m \tag{68}$$

$${}_0^C D_t^\alpha \upsilon_m = \bar{p}_m\lambda_m\zeta_m + \frac{1}{2}\bar{p}_m\zeta_m^2 - \bar{\beta}_m\upsilon_m \tag{69}$$

$${}_0^C D_t^\alpha \Theta = p_{11}\zeta_m\varpi_m - \beta_{11}\Theta \tag{70}$$

$${}_0^C D_t^\alpha \varepsilon = p_{22}\zeta_m\tanh\left(\frac{\zeta_m}{q_3}\right) - \beta_{22}\varepsilon \tag{71}$$

where $c_m > 0$, $\beta_m > 0$, $\bar{\beta}_m > 0\beta_{11} > 0$ and $\beta_{22} > 0$ are design parameters.

By substituting (67)–(71) into (66), one has

$$\begin{aligned} w_0^C D_t^\alpha V \le {}& \sum_{j=1}^m \left(-c_j\zeta_j^2 - \frac{\beta_j}{p_j}\tilde{W}_j W_j - \frac{\bar{\beta}_j}{\bar{p}_j}\tilde{\upsilon}_j\upsilon_j\right) \\ & + \sum_{j=1}^{m-1}(\nu_j\left(\frac{\nu_j}{2} - \frac{\nu_j}{\sigma_j} + D_j(.)\right) - \frac{\bar{c}}{\underline{p}}r \\ & + \frac{\theta^*\beta_{11}}{p_{11}}\tilde{\Theta}\Theta + \rho_m + \frac{\beta_{22}}{p_{22}}\tilde{\varepsilon}\varepsilon \end{aligned} \tag{72}$$

where $\rho_m = \rho_{m-1} + q\upsilon_m^* + \frac{\tau_m^{*2}}{2} + d_m(t_0,t) + s + \omega\theta^* + 0.2785\varepsilon^* q_3$.

Theorem 1. *The research object is FONS(1) with unmodelled dynamics and actuator failure. Suppose Assumptions 1–5 satisfied, the devised actual controller (63), the virtual controllers (27), (44) and (62) and the FO parameter adaptation laws (28), (29), (45), (46), (68), (69), (70) and (71) are adopted. Then the control algorithm can ensure two things, one is that all the signals of the closed-loop system are bounded, the other is to make the tracking errors as small as possible.*

Proof: Under Assumption 3 with a constant $\pi > 0$, the sets Ξ_0, and $\Xi = \{\sum_{i=1}^m \frac{\zeta_i^2}{2} + \sum_{i=1}^m \frac{\tilde{W}_i^2}{2p_i} + \sum_{i=1}^m \frac{\tilde{\upsilon}_i^2}{2\bar{p}_i} + \frac{r}{\underline{p}} + \sum_{i=1}^{m-1}\frac{\nu_i^2}{2} + \frac{\theta^*}{2p_{11}}\tilde{\Theta}^2 + \frac{1}{2p_{22}}\tilde{\varepsilon}^2 \le \pi\}$ are compact sets. Consequently $\Xi_0 \times \Xi$ is a compact set. Therefore, $\exists K_i > 0$ such that $|D_i| \le K_i$.

By the Young inequality, we get

$$\tilde{W}_j W_j + \tilde{\upsilon}_j\upsilon_j \le -\frac{1}{2}\tilde{W}_j^2 + \frac{1}{2}W_j^{*2} - \frac{1}{2}\tilde{\upsilon}_j^2 + \frac{1}{2}\upsilon_j^{*2} \tag{73}$$

$$\nu_b D_b + \tilde{\Theta}\Theta + \tilde{\varepsilon}\varepsilon \le \frac{\nu_b^2}{2} + \frac{K_b^2}{2} - \frac{\tilde{\Theta}^2}{2} + \frac{\Theta^{*2}}{2} - \frac{\tilde{\varepsilon}^2}{2} + \frac{\varepsilon^{*2}}{2} \tag{74}$$

From (72)–(74), one can obtain

$$\begin{aligned} {}_0^C D_t^\alpha V \le {}& \sum_{j=1}^m \left(-c_j\zeta_j^2 - \frac{\beta_j}{2p_j}\tilde{W}_j^2 - \frac{\bar{\beta}_j}{2\bar{p}_j}\tilde{\upsilon}_j^2\right) \\ & + \sum_{j=1}^{m-1}\left(\nu_j^2\left(1 - \frac{1}{\sigma_j}\right)\right) \\ & - \frac{\theta^*\beta_{11}\tilde{\Theta}^2}{2p_{11}} - \frac{\beta_{22}\tilde{\varepsilon}^2}{2p_{22}} + \rho - \frac{\bar{c}}{\underline{p}}r \end{aligned} \tag{75}$$

Define $\eta = \min\{2c_j, \beta_j, \bar{\beta}_j, \bar{c}, j = 1, 2, \ldots, m; 2/\sigma_j - 2, j = 1, 2, \ldots, m-1; \beta_{11}, \beta_{22}\}$ $\rho = \sum_{j=1}^{m}\left(\frac{\beta_j}{2p_j}W_j^{*2} + \frac{\bar{\beta}_j}{2\bar{p}_j}\upsilon_j^{*2}\right) + \sum_{j=1}^{m-1}\frac{K_j^2}{2} + \frac{\theta^*\beta_{11}\Theta^{*2}}{2p_{11}} + \frac{\beta_{22}\varepsilon^{*2}}{2p_{22}} + \rho_m$. Then, (75) can be rewritten as

$$ {}_0^CD_t^\alpha V \leq -\eta V + \rho \tag{76}$$

Using [7, 12] and [36], from (76), we have

$$ {}_0^CD_t^\alpha V + \Phi(t) = -\eta V + \rho \tag{77}$$

where $\Phi(t) > 0$.

Taking Laplace transform for both sides of (77), it will give

$$\begin{aligned} V(s) &= \frac{s^{\alpha-1}V(0)}{s^\alpha + \eta} + \frac{\rho}{s(s^\alpha + \eta)} - \frac{\Phi(s)}{s^\alpha + \eta} \\ &= \frac{s^{\alpha-1}V(0)}{s^\alpha + \eta} + \frac{s^{\alpha-(\alpha+1)}\rho}{s^\alpha + \eta} - \frac{\Phi(s)}{s^\alpha + \eta} \end{aligned} \tag{78}$$

Then, taking the inverse Laplace transform for the Equation (78), we have

$$\begin{aligned} V(t) &= E_{\alpha,1}(-\eta t^\alpha)V(0) + t^\alpha E_{\alpha,\alpha+1}(-\eta t^\alpha)\rho \\ &\quad - \Phi(t) * t^{\alpha-1}E_{\alpha,\alpha}(-\eta t^\alpha) \end{aligned} \tag{79}$$

where $*$ is the convolution operator. Since $\Phi(t)$and $t^{\alpha-1}E_{\alpha,\alpha}(-\eta t^\alpha)$ are non-negative functions, the term $\Phi(t) * t^{\alpha-1}E_{\alpha,\alpha}(-\eta t^\alpha) \geq 0$ in (67). Therefore, we get

$$ V(t) \leq E_{\alpha,1}(-\eta t^\alpha)V(0) + t^\alpha E_{\alpha,\alpha+1}(-\eta t^\alpha)\rho \tag{80}$$

Using Lemma 1 and we have

$$ |t^\alpha E_{\alpha,\alpha+1}(-\eta t^\alpha)\rho| \leq \frac{\rho t^\alpha d}{1 + |\eta t^\alpha|} \leq \frac{\rho d}{\eta} \tag{81}$$

where d is a positive constant. Thus

$$ V(t) \leq V(0)E_{\alpha,1}(-\eta t^\alpha) + \frac{\rho d}{\eta}, \quad t \geq 0 \tag{82}$$

Similarly, we get

$$ |E_{\alpha,1}(-\eta t^\alpha)| \leq \frac{r}{1 + \eta t^\alpha} \tag{83}$$

Finally, (79) and the tracking error are

$$ V(t) \leq V(0)\frac{r}{1 + \eta t^\alpha} + \frac{\rho d}{\eta}, \ t \geq 0 \tag{84}$$

$$ \frac{1}{2}\zeta_1^2 \leq V(t) \leq V(0)\frac{r}{1 + \eta t^\alpha} + \frac{\rho d}{\eta} \tag{85}$$

From (85), we further have

$$ |\zeta_1| \leq \sqrt{\frac{2rV(0)}{1 + \eta t^\alpha} + \frac{2\rho d}{\eta}} \tag{86}$$

It can be concluded from (84) that the controlled object is stable. For (86), as $t$goes to infinity, we get $\lim_{t\to\infty}|\zeta_1| \leq \sqrt{\frac{2\rho d}{\eta}}$. Therefore, it follows that all signals of the closed-loop system are bounded, and the tracking error can achieve satisfactory performance and that completes the proof.

4 | SIMULATION STUDY

In this section, an example is given to illustrate the effectiveness of the proposed control algorithm.

Example: Consider the following FONS:

$$\begin{aligned} {}_0^CD_t^\alpha z &= -z + x_1^2 \\ {}_0^CD_t^\alpha x_1 &= x_1\sin(x_2) + x_2 + zx_1\sin(x_2) \\ {}_0^CD_t^\alpha x_2 &= \sin(x_1)x_2 + u_1 + u_2 + zx_1\sin(x_2) \\ y &= x_1 \end{aligned} \tag{87}$$

where $\alpha = 0.98$, $f_1(X) = x_1\sin(x_2)$, $f_2(X) = \sin(x_1)x_2$, $q(z, X) = -z + x_1^2$, $H_1(z, X) = zx_1\sin(x_2)$ and $H_2(z, X) = zx_1\sin(x_2)$.

The following actuator faults model can be expressed as:

$$ u_1^F = \begin{cases} u_1, & \text{if } t \geq 20 \\ 12, & \text{if } t < 20 \end{cases} \tag{88}$$

$$ u_2^F = \begin{cases} u_2, & \text{if } t \geq 20 \\ 0.9u_2, & \text{if } t < 20 \end{cases} \tag{89}$$

To make Assumption 4 hold for the z-system in (87), we can choose $V_z(z) = z^2$. Then, by the inequality ${}_0^CD_t^\alpha(x^\tau(t)x(t))/2 \leq x^\tau(t){}_0^CD_t^\alpha x(t)$ in [29] and Young's inequality, we get

$$\begin{aligned} {}_0^CD_t^\alpha V_z(z) &\leq z(-z + x_1^2) \\ &\leq -0.5z^2 + x_1^4 + \frac{1}{4} \end{aligned} \tag{90}$$

Taking $\alpha_1(|z|) = 0.5z^2$, $\alpha_2(|z|) = 1.5z^2$, $c_i = 0.5$, $\gamma(|x_1|) = x_1^4$ and $d = \frac{1}{4}$. Then, Assumption 4 holds.

By selecting $\bar{c} = 0.2 \in (0, c)$, a FO dynamic signal has the following form

$$ {}_0^CD_t^\alpha r = -0.2r + x_1^4 + \frac{1}{4} \tag{91}$$

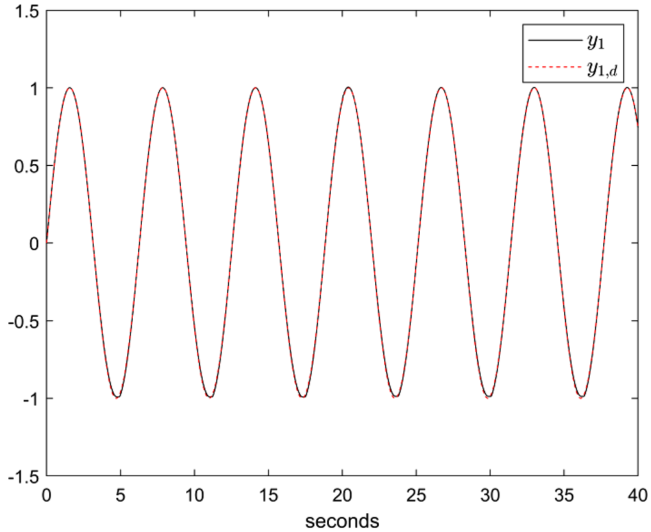

FIGURE 1 The trajectories of y_d and y

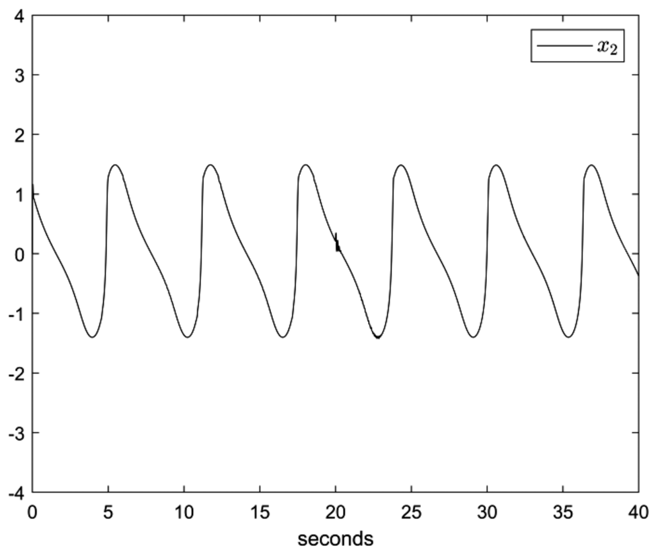

FIGURE 2 The trajectory of x_2

The simulation uses five nodes for each in dimensions of $w^\tau\psi(X)$, centre $\eta_i = 0$, width $\mu_i = 4$, and evenly spaced in the interval $[-2, 2] \times [-2, 2]$.

Given the desired reference signals as $y_{i,d} = \sin(t)$, choose the design parameters as $c_1 = 63, c_2 = 68, p_i = 0.2, \bar{p}_i = 0.1, \underline{p} = 1$, $\beta_i = \bar{\beta}_i = 0.1$, $p_{ii} = 0.1$, $\beta_{ii} = 0.1\sigma_1 = 0.001$, $q_1 = q_2 = q_3 = 2$ and $\omega = 6$. The initial conditions of $X(0) = [x_1(0), x_2(0)]^\tau = [0.002, 0.002]^\tau$, $z(0) = 0.5$, $r(0) = 2$, $\varepsilon(0) = 2$ and other variables are equal to zero.

Figures 1–4 reveal the simulation results. Figure 1 depicts the trajectories of desired reference signal y_d and the system output y. Figure 2 describes the trajectories of states x_2. The trajectories in Figure 3 describe the tracking errors ζ_1. The trajectories of u_1 and u_2 are shown in Figure 4.

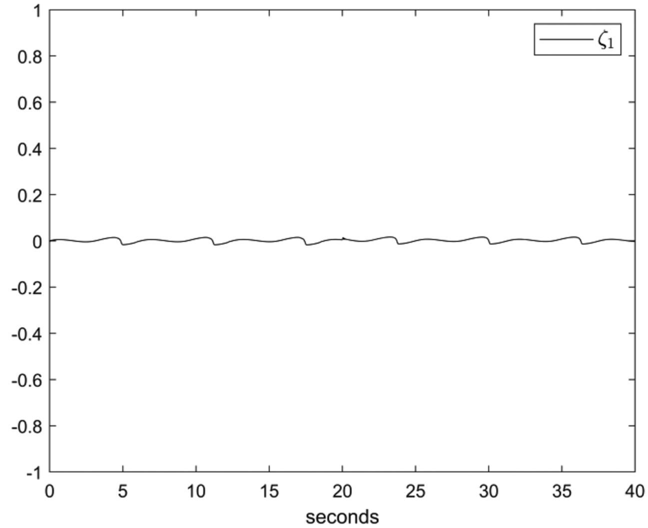

FIGURE 3 The trajectory of ζ_1

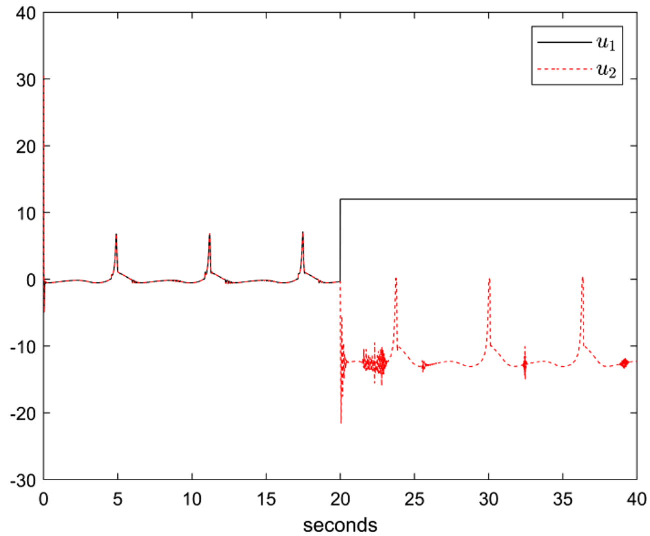

FIGURE 4 The trajectories of u_1 and u_2

Figures 1–4 clearly show that the put forward control algorithm could make all the variables involved in the closed-loop system bounded. More importantly, the tracking errors could converge to a small neighbourhood of the origin.

5 | CONCLUSIONS

This study has investigated the tracking control problem for FONS with unmodelled dynamics and actuator faults. To deal with unknown non-linear continuous functions and unmodelled dynamics, we adopt NNs and FO dynamic signals. According to the bound estimation, the put forward FTC method tolerates actuator faults without knowing any information. Motivated by the adaptive backstepping recursive design algorithm with the

FO Lyapunov stability criterion, a stable NNs adaptive FTC technique has been proposed. The main superiority and novelties of the proposed control algorithm are that it can not only ensure the stability of the controlled system but also reduce the tracking error. Moreover, applying the proposed control algorithm to the numerical example can indicate the capability of the method. Further research will focus on fractional-order multi-agent systems based on the proposed approach.

ACKNOWLEDGEMENT

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CONFLICT OF INTEREST

The authors declare that there is no conflict of interest, I do not have any possible conflicts of interest.

DATA AVAILABILITY STATEMENT

Some or all data, models, or codes that support the findings of this study are available from the corresponding author upon reasonable request.

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