State Feedback Controller Tuning for Liquid Slosh Suppression System Utilizing LQR-LMI Approach

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***Abstract*—This paper presents a tuning constraint optimization approach in state feedback controller for liquid slosh suppression system. A suboptimal LQR method is employed to obtain the optimal gain parameters in minimizing the selected cost function. Due to complexity of the nonlinear slosh system, a partial linearization method was first performed to obtain its linear state space representation. Due to the presence of the large steady-state error caused by the implementation of only the state feedback gains, an additional integral term has also been introduced to mitigate its effects. A comparative assessment on the system performance is investigated between regular LQR and LQR-LMI control algorithms. The presented results indicated that the LQR-LMI exhibited better transient response performance as compared to the regular LQR for the case of moving the cart to its intended final position while ensuring the slosh motion is suppressed to a minimum angle.**

Keywords*—slosh suppression, state feedback controller, integral action, suboptimal LQR, Linear Matrix Inequality (LMI)

I. Introduction

The container with filled liquid would create a slosh motion while it is being moved in certain direction [1]. Some common industrial applications that involve the slosh phenomenon can be found in the heavy-duty industries sector such as in liquid cargo carriers, molten metal or beverage transportation and space launch system [2]. It is desirable that the system of the filled container to be moved and reach the desired final position as fast as possible. However, that fast motion may induce slosh that comes in oscillated form and may consequently pose danger to the overall safety [3], [4]. Besides, the instability might occur due to the changes of dynamic system structure that later degrade the effectiveness of the system to run smoothly. Hence, it is essential to design a controller that suppress this sloshing behavior while moving the container, to ensure the stability and functionality of the complete system are in top-notch conditions.

Various control strategies have been explored by researchers to optimize the liquid slosh suppression system that can be found from the literature surveys. Mostly, those of previous works implemented the closed-loop based approach in designing control system, which is much stable and less sensitive to noise as compared to the open-loop system. For instance in [5], the authors shows promising results by implementing Sliding Mode control based on SMO through PID scheme. Meanwhile the works in [6] and [7], focused on applying fuzzy logic controller based on rule table and membership function knowledge to control the sloshing effects in the space craft application. Furthermore in [8] and [9], the authors presented the H-infinity approach by locating the parameters gain in certain constrain. The results in [10], [11], [12] shows a capability of data-driven technique based on Simultaneous perturbation stochastic approximation (SPSA), Exponent-based Simulated Kalman Filter (EbSKF) and Single input fuzzy logic controller (SIFLC) to tune the system without required any explicit form of the objective function.

The Linear Quadratic Regulator (LQR) is known as an ideal approach to provide practical parameters gain [13]. This technique eliminates the transient error signal based on control weighting matrix for tradeoff in cost function [14]. In this study, the LQR approach is extended to utilize the suboptimal Linear Quadratic Regulator-Linear Matrix Inequality (LQR-LMI) method as adopted from [15] to obtain the state feedback parameters gain to control the liquid slosh suppression system. Wherein, the stability of the controlled system is achieved by placing the closed-loop poles location on the left side of the complex s-plane. In addition, to reach to a good transient performance, an integral term is added to provide zero steady state error for the output parameters. In this study, the performance analysis featuring the MATLAB simulation software was performed by comparing the closed-loop performance of both LQR and suboptimal LQR-LMI method in terms of minimizing the cost function formulated based on the Integral Square Error (ISE).

The rest of this paper is organized as follows. In Section II, the complete description on the methodology of the proposed techniques is presented. This includes the development of mathematical model structure and control system of state feedback controller with integral term by utilizing suboptimal LQR. Next in section III, a numerical example is presented to elucidate our proposed method. Here, the output response and performance of the closed loop system in terms of cart's trajectory tracking and the liquid slosh suppression are discussed. Finally, the main conclusion is stated in section IV.

II. Methodology

In this section, the nonlinear modeling of the liquid slosh suppression system is first presented. Then, the linearization procedures that were undertaken as part of designing the state feedback controller with integral term is briefly explained. Next, the state feedback controller tuning utilizing both the LQR and suboptimal LQR-LMI algorithm will be discussed.

A. Nonlinear system modeling and linear State Space representation

A partially filled liquid container that performing rectilinear motion is considered as the controlled plant. The system performing in lateral slosh fundamental mode which is highly complex [2]. Here, a simple mechanism is adapted in pendulum model to eliminate the complexity of lateral slosh. The slosh mass, m represented in mass pendulum while other mass that does not involved with slosh motion represented as rigid mass. The applied force, f through the model system exhibit pendulum angular motion, θ by movement of pendulum mass, which is portrayed as the angular motion of slosh motion. The illustrated comparison between the equivalent mechanical model of a partially filled container of the slosh system, and the simple pendulum model are shown in Fig. 1.

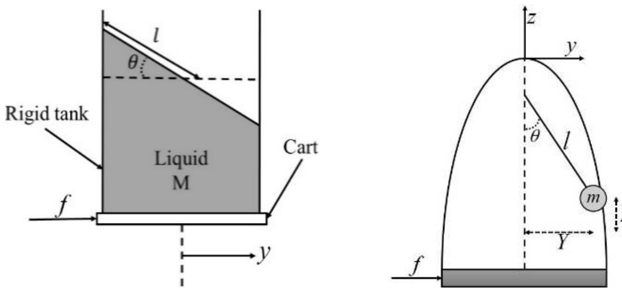

Fig. 1. Partially filled container model and simple pendulum model of slosh system

Based on schematic model depicted in Fig. 1, the Euler-Lagrage equation is used to derive the dynamics equations of the given system. The equation of motion of the nonlinear system is expressed as:

$$M\ddot{y} + ml\cos\theta\ddot{\theta} - ml\dot{\theta}^2\sin\theta = f \tag{1}$$

$$ml\cos\theta\ddot{y} + ml^2\ddot{\theta} + d\dot{\theta} + mgl\sin\theta = 0 \tag{2}$$

where the parameters in (1) and (2) are defined as follows :

- M : Mass of tank and liquid
- m : Mass of the Liquid
- l : Hypotenuse length of the slosh
- g : Gravitational constant
- d : Damping coefficient
- f : Applied force for translation motion of the cart
- θ : Angle motion
- y : Displacement of the cart

The equations of (1) and (2) illustrate a highly second order under actuated system, which is complex and difficult to be tuned. Therefore, a linear system structure to simplify the controller design process and analysis has been considered. By partial linerization [1], one could transform (1) and (2) to its state space representation as (3) and (4) to depict the linear dynamic system of.

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{3}$$

$$y(t) = Cx(t) \tag{4}$$

where $x(t) = \begin{bmatrix} y(t) & \dot{y}(t) & \theta(t) & \dot{\theta}(t) \end{bmatrix}^T \in \mathbb{R}^4$ is the state vector and $u(t) \in \mathbb{R}$ is the input vector. Meanwhile, $A \in \mathbb{R}^{4\times4}$, $B \in \mathbb{R}^4$, and $C \in \mathbb{R}^{1\times4}$ are the system matrix, input and output vectors of the linear system, where they are defined as in (5), (6), and (7), respectively, as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{-g\zeta}{l} & \dfrac{-d}{ml^2} \end{bmatrix} \tag{5}$$

$$B = \begin{bmatrix} 0 & 1 & 0 & \dfrac{-\cos\theta}{l} \end{bmatrix}^T \tag{6}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

where $\zeta = \dfrac{\sin\theta}{\theta}$.

B. Controller Design

We consider the controlled system design to implement the state feedback controller with integral action in closed-loop path system as shown in Fig. 2. Generally, the desired design specification is fulfilled by seeking the optimal parameters gains through relocating the poles of the closed-loop system in the complex s-plane.

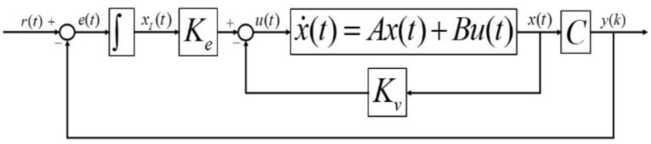

Fig. 2. State feedback controller with integral term

The parameter $K_v \in \mathbb{R}^{1\times4}$ depicted in Fig. 2 is the state feedback controller's gains. The implementation of only the state feedback gains does not guarantee the steady-state error defined by $e_{ss} = \lim_{t\to\infty} e(t)$ to reach zero as $t \to \infty$. Therefore, it is essential to mitigate this error through some other control action. Hence, an additional integrator and integral gain of $K_e \in \mathbb{R}$ as shown in Fig. 2 is imposed. The introduction of the additional term eventually reshapes the overall dynamic and increases the number of the state variables. Let $x_I \in \mathbb{R}$ be denoted as the integral state and its time derivative satisfies the following notation:

$$\dot{x}_I = -Cx(t) + r(t) \tag{8}$$

The integral term is added to dynamic system (3), and a new linear system in state space representation is formed as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_I(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \tag{9}$$
$$e(t) = r(t) - Cx(t)$$

Note that the system defined in (9) is a type of a "servo" control problem. Assuming the desired condition that as $t \to \infty$, all the states' time derivative shall converge to zero. Hence, based on (9), equation (10) may be formulated as:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(\infty) \\ x_I(\infty) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(\infty) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(\infty) \tag{10}$$
$$e(\infty) = r(\infty) - Cx(\infty)$$

Then, by subtracting equation (9) from (10) to form a new dynamic at the steady state in the type of "regulator" control problem as follows:

$$\dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}\tilde{u}(t) \tag{11}$$

$$\tilde{e}(t) = \tilde{C}\tilde{x}(t) \tag{12}$$

$$y(t) = \tilde{C}\tilde{x}(t) \tag{13}$$

where the new system matrix, input and output vectors are now redefined as

$$\tilde{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} -C & 0 \end{bmatrix},$$
$$\tilde{x}(t) = \begin{bmatrix} x(t) - x(\infty) \\ x_I(t) - x_I(\infty) \end{bmatrix}, \quad \tilde{e}(t) = e(t) - e(\infty).$$

By using the new system, we may express the modified control law as

$$\tilde{u}(t) = K\tilde{x}(t) = \begin{bmatrix} -K_v & K_e \end{bmatrix} \tilde{x}(t). \tag{14}$$

C. Linear Quadratic Regulator (LQR)

The aim of designing the state feedback controller by LQR is to obtain the optimal gain for the control law (14) which minimizes the performance index defined by

$$J = \int_0^\infty (\tilde{x}^T Q \tilde{x} + \tilde{u}^T R \tilde{u})\, dt \tag{15}$$

The parameters of $Q = Q^T \geq 0 \in \mathbb{R}^{5\times5}$, and $R = R^T \geq 0 \in \mathbb{R}$ are the weighted matrices of state and control system, respectively. Hence, the control action that minimize (15) can be related as

$$\tilde{u}(t) = K\tilde{x}(t) = R^{-1}\tilde{B}^T P \tilde{x}(t). \tag{16}$$

The positive definite matrix $P = P^T > 0 \in \mathbb{R}^{5\times5}$ in (16) is obtained by computing the algebraic Riccati equation (ARE) of

$$\tilde{A}^T P + P\tilde{A} + P\tilde{B}R^{-1}\tilde{B}^T P + Q = 0 \tag{17}$$

with the minimal performance index defined by $J_{\min} = \tilde{x}(0)^T P \tilde{x}(0)$, where $\tilde{x}(0)$ is the initial states vector. To find the solution of (17), we may employ the existing Matlab command of '`lqr`'.

D. Suboptimal LQR based on LMI (LQR-LMI)

In our proposed method, the LQR is modified into LMI constraint optimization method, in which LMI is known as powerful design tool for solving many convex problems [16]. Adopting similar approach as in [15], suboptimal cost $\gamma \geq J_{\min}$ is computed through LQR-LMI method instead of $J_{\min}$. Here, the suboptimal problem is solved based on choice of $\gamma \in \mathbb{R}$, $x(0)$, $Q = Q^T > 0$, and $R = R^T > 0$. By this approach, the standard LQR problem can be recast into the suboptimal LQR-LMI as an optimization problem over $\hat{P} = P^{-1} \in \mathbb{R}^{5\times5}$ and $Y \in \mathbb{R}^{1\times5}$ as

$$\min_{\hat{P},\, Y} x^T(0)\hat{P}^{-1}x(0) \tag{18}$$

subject to

$$\begin{bmatrix} \tilde{A}\hat{P} + \hat{P}\tilde{A}^T + \tilde{B}Y + Y^T\tilde{B}^T & \hat{P} & Y^T \\ \hat{P} & -Q^{-1} & 0 \\ Y & 0 & -R^{-1} \end{bmatrix} \leq 0,$$
$$\hat{P} > 0. \tag{19}$$

Based on suboptimal cost, the objective function (18) is reformulated as

$$\tilde{x}(0)^T P \tilde{x}(0) = \tilde{x}(0)^T \hat{P}^{-1} \tilde{x}(0) \leq \gamma \tag{20}$$

Then, Schur's complement is used to express the (20) in LMI constraint form of

$$\begin{bmatrix} \gamma & \tilde{x}(0)^T \\ \tilde{x}(0) & \hat{P} \end{bmatrix} \geq 0, \tag{21}$$

where the optimal gains of the control law are obtained based on

$$K = Y\hat{P}^{-1}. \tag{22}$$

The following algorithm and the partial snapshot of Matlab codes (as depicted in Fig. 3) have been used to compute the state feedback gain (22):

Step 1: Determine parameter matrices of $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$

Step 2: Choose an appropriate value of γ, initial state $x(0)$ and suitable weights $Q > 0$, and $R > 0$.

Step 3: Solve (19) and (21) to obtain parameters of Y and $\hat{P}$.

Step 4: Solve (22) to obtain optimal gains of K.

```
setlmis ([])
P = lmivar(1,[5 1]);
Y = lmivar(2,[1 5]);
S1 = newlmi;
lmiterm([S1 1 1 P],Ai,1,'s')%AP+PA'
lmiterm([S1 1 1 Y],Bi,1,'s')%BY+Y'B'
lmiterm([S1 1 2 P],1,1)%P
lmiterm([S1 1 3 -Y],1,1)%Y'
lmiterm([S1 2 1 P],1,1)%P
lmiterm([S1 2 2 0],inv(-Q))%-Q^-1
lmiterm([S1 3 1 Y],1,1)%Y
lmiterm([S1 3 3 0],inv(-R))%-Q^-1
lmiterm([-S1 4 4 P],1,1)%P>0
lmiterm([-S1 5 5 0],gamma)%gamma
lmiterm([-S1 5 6 0],x0')%x(0)'
lmiterm([-S1 6 5 0],x0)%x(0)
lmiterm([-S1 6 6 P],1,1)%P
LMIs = getlmis;
[tmin,xopt] = feasp(LMIs);
P1 = dec2mat(LMIs,xopt,P)
Y1 = dec2mat(LMIs,xopt,Y)
K = Y1*inv(P1)
```

Fig. 3. Partial snapshot of Matlab code to compute (22).

E. Integral Square Error (ISE)

To evaluate the effectiveness of the tuning methods, we consider the performance index in the form of integral square error evaluated in time interval of $[t_0,t_F]$ defined by

$$ISE=\int_{t_0}^{t_F}\left(e_1^2(t)+e_2^2(t)\right)dt \tag{23}$$

where $e_1(t)=r(t)-Cx(t)=r(t)-y(t)$ is the error between desired and actual cart's displacement, and $e_2(t)=0-\theta(t)$ is the error related to the slosh motion. The selection of the equation (23) will provide the quantitative analysis of the controlled system with state feedback controller, designed based on LQR and suboptimal LQR methods. Thus, the best approach is determined based on the comparison of index that reaches to the extremum value [17], i.e., $\min ISE\geq 0$.

III. Result Simulation

A numerical example is presented in this section to elucidate the feasibility of the proposed method. We adopted the system's parameters from [9] as tabulated in Table 1. Based on this configuration, the performance analysis has been conducted by employing the Matlab/Simulink simulation package.

TABLE I. Parameters of the Slosh Model System

Parameters	Value	Unit
M	6.0	kg
m	1.32	kg
l	0.052126	m
g	9.18	m/s^2
d	3.0490×10^{-4}	kgm^2/s

The linear system dynamic of (5),(6) and (7) were then obtained as

$$A=\begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&-153.8447&-0.0850\end{bmatrix},\quad B=\begin{bmatrix}0\\1\\0\\-9.5921\end{bmatrix},\quad \text{and}$$

$C=\begin{bmatrix}1&0&0&0\end{bmatrix}$, respectively.

For the selection of the reference input, we considered the trajectory of the desired signal in the form of

$$r(t)=\begin{cases}1, & 0\leq t\leq 5,\\ 0, & 5\leq t\leq 10,\\ 0.5, & 10\leq t\leq 20.\end{cases} \tag{24}$$

In the LQR method, the matrices Q and R satisfying the conditions of $Q=Q^T>0$, and $R=R^T>0$ are arbitrarily chosen such as

$$Q=\begin{bmatrix}40&0&0&0&0\\0&1&0&0&0\\0&0&70&0&0\\0&0&0&1&0\\0&0&0&0&110\end{bmatrix}, \tag{25}$$

$$R=1\times10^{-3}. \tag{26}$$

Utilizing (25) and (26), the matrix P was obtained by the Matlab algorithm with the instruction command of $[\mathtt{K1},\mathtt{P},\mathtt{E}]=\mathtt{lqr}(\mathtt{A},\mathtt{B},\mathtt{Q},\mathtt{R})$. Instead of assigning K1 as the state feedback gain, we computed the matrix K by equation (16) using the previously obtained matrix P. Hence, by this convention, the optimal gain parameters were obtained as

$$K=\begin{bmatrix}364.8779 & 140.4077 & -411.9620 & -18.5942 & -331.6625\end{bmatrix}.$$

Next, in the suboptimal LQR-LMI method, we chose initial states of $\tilde{x}(0)=\begin{bmatrix}0&0&0&0&0\end{bmatrix}^T$ and $\gamma=1$. Hereby, the variables $\hat{P}$ and Y were determined based on LMI constraints of (19) and (21) with the appropriate indexes. By computing (21) (see Fig. 3 for the partial snapshoot of Matlab code), we obtained the optimal controller parameters as

$$K=\begin{bmatrix}1.1585 & 0.3912 & -0.9356 & -0.0074 & 1.2626\end{bmatrix}\times10^3.$$

The trajectory of the cart's position and velocity are depicted in in Figs. 4 and 5, respectively. In both methods, the cart moves to its intended final position in smooth response with zero overshoot and minimum steady state error as shown in Fig. 4. Furthermore, it could also be observed that the output response with the state feedback controller tuned by the suboptimal LQR-LMI, had exhibited faster response than the one tuned by regular LQR. This finding can be verified where in terms of settling time, the suboptimal LQR-LMI approach depicted faster response than LQR by a difference within 0.7285 seconds. On the other hand, suboptimal LQR-LMI required 0.52 seconds of rise time as compared to LQR with 0.58 seconds. The system performance based on the output response of the cart's linear displacement is summarized in Table II.

TABLE II. CART MOTION SYSTEM PERFORMANCE

System performance	LQR	LQR-LMI
Settling time (s)	12.7699	12.0414
Rise time (s)	0.5876	0.5227
Steady-state error	0	0
Overshoot (%)	0	0

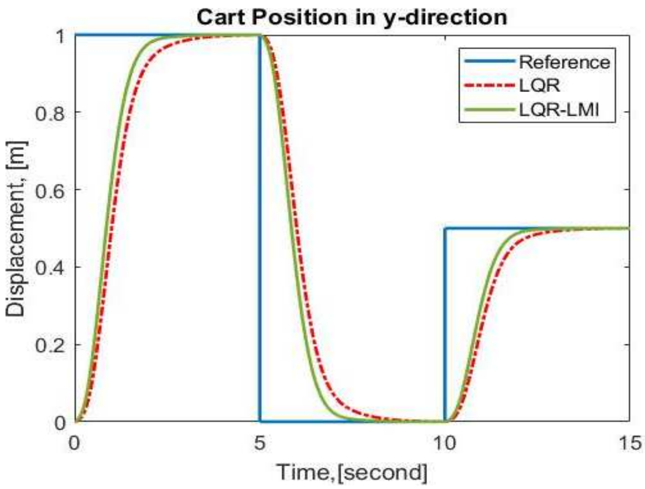

Fig. 4. Cart's displacement in y-direction response.

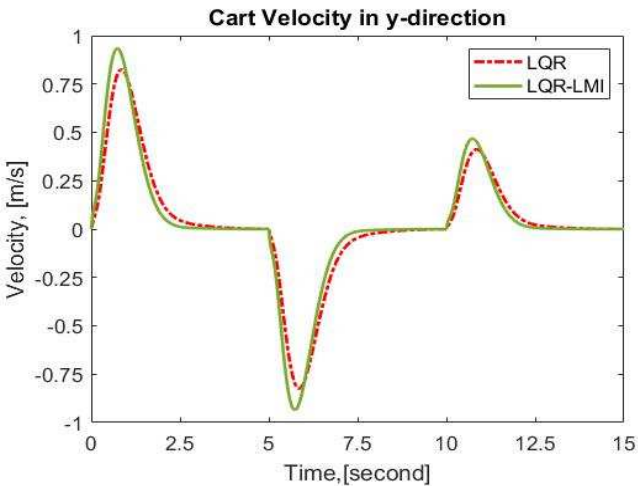

Fig. 5. Cart's velocity response

The slosh motion representation in terms of the slosh angular displacement is presented in Fig. 6. Meanwhile, its angular velocity is exhibited in the subsequent plot of Fig.7. Based on these results, it can be vividly observed that the slosh angular displacement and velocity of the controlled system tuned by both methods only yield small oscillating signal which implies that the slosh is at the minimum level as the cart is in motion. However, by analyzing the performance of the system in terms of ISE, it was found the suboptimal LQR-LMI produced ISE = 1.51515 as compared to regular LQR which produced ISE =1.7793. This is an indication that the state feedback controller tuned by the suboptimal LQR-LMI produced better and faster output response than the one tuned by the regular LQR. The system performance for the slosh motion response is summarized in Table III.

TABLE III. SLOSH MOTION PERFORMANCE

System performance	LQR	LQR-LMI
Settling time (s)	12.5328	12.2438
Maximum angle (rad)	0.11	0.15
ISE	1.7793	1.5115

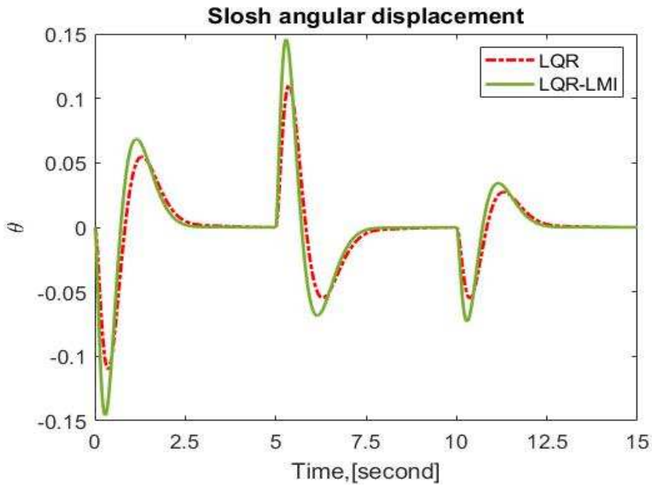

Fig. 6. Slosh angular displacement.

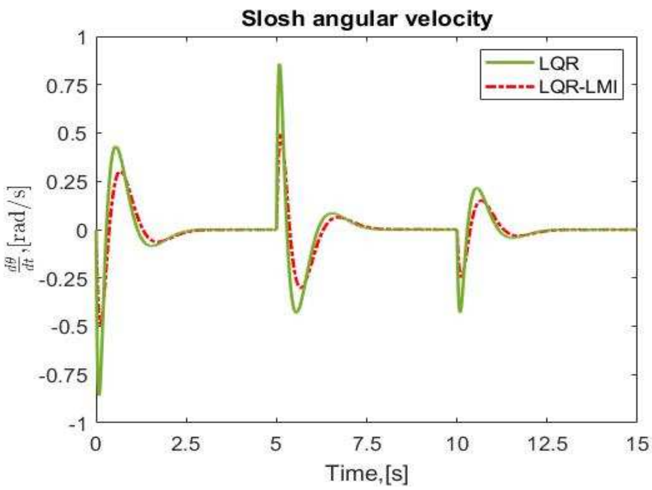

Fig. 7. Slosh angular velocity.

IV. CONCLUSION

In this paper, the state feedback controller tuning for a liquid slosh suppression system using the regular LQR and suboptimal LQR based on LMI constraints have been presented. The findings have been obtained through numerical analysis employing the Matlab/Simulink simulation package. Based on the presented results, it can be concluded that the controlled system tuned by the suboptimal LQR-LMI exhibited better transient response performance as compared to the one tuned by the regular LQR. The results have been validated for both cart linear movement and the liquid slosh motion.

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