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H_∞ output tracking control for a class of switched LPV systems and its application to an aero-engine model

Dong Yang and Jun Zhao*,†

State Key Laboratory of Synthetical Automation for Process Industries, College of Information Science and Engineering, Northeastern University, Shenyang 110819, China

SUMMARY

This paper aims to investigate the problem of H_∞ output tracking control for a class of switched linear parameter-varying (LPV) systems. A sufficient condition ensuring the H_∞ output tracking performance for a switched LPV system is firstly presented in the format of linear matrix inequalities. Then, a set of parameter and mode-dependent switching signals are designed, and a family of switched LPV controllers are developed via multiple parameter-dependent Lyapunov functions to enhance control design flexibility. Even though the H_∞ output tracking control problem for each subsystem might be unsolvable, the problem for switched LPV systems is still solved by the designed controllers and the designed switching law. Finally, the effectiveness of the proposed control design scheme is illustrated by its application to an H_∞ speed adjustment problem of an aero-engine. Copyright © 2016 John Wiley & Sons, Ltd.

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KEY WORDS: switched linear parameter-varying system; H_∞ output tracking control; parameter and mode-dependent switching signal; multiple parameter-dependent Lyapunov functions; aero-engine control

1. INTRODUCTION

Switched systems are composed of an indexed family of subsystems described by continuous or discrete-time dynamics and a rule governing the switching between them, which usually depends on time or states, or other variables. Many real processes and systems can be modeled as switched systems because of the various jumping parameters and changing environmental factors [1, 2]. The widespread applications of switched systems are also motivated by increasing performance requirements in control [3, 4]. Many problems of switched systems are not solvable by a single controller, but they can be solved by switching controllers [5, 6]. Afterwards, several approaches have been proposed in the study of switched systems, such as common Lyapunov function, single Lyapunov function, multiple Lyapunov functions [7], and so forth. In particular, the multiple Lyapunov function approach often offers greater freedom for demonstrating control synthesis of switched systems and has proven to be a powerful tool for finding a desirable switching law. Various switching laws have been proposed for switched systems, and they can be categorized mainly as state dependent [8, 9] or time driven [7, 8]. Recently, the improved state-dependent [10] and average dwell-time [11] switching control strategy is found.

However, most of the current works are focused on switched systems whose subsystems are described by linear time-invariant (LTI) dynamics [12, 13]. Normally, systems in practice are usually nonlinear. In order to study switched nonlinear systems with aid of LTI system techniques, linear parameter-varying (LPV) subsystems are sometimes needed, which can describe an important

*Correspondence to: Jun Zhao, State Key Laboratory of Synthetical Automation for Process Industries, College of Information Science and Engineering, Northeastern University, Shenyang 110819, China.
†E-mail: zhaojun@mail.neu.edu.cn

class of nonlinear systems by using the time-varying parameters [14–18]. Meanwhile, a few works investigate the control problem for switched LPV systems. On one hand, some contributions investigate the switched LPV systems where the switching signals are parameter dependent but mode independent. The control techniques for switched LPV systems under hysteresis and average dwell-time logics are developed in [15]. The controller state reset is dealt with, and two switching logics are proposed in [16]. On the other hand, a small number of valuable results focus on the switched LPV systems in which the switching signals are mode dependent but parameter independent. In these contributions, both the model reduction problem [17] and the H_∞ control problem [18] of asynchronously switched LPV systems have been solved via average dwell time method. It should be noted that, in the existing contributions on the study of switched LPV systems, all subsystems are required to be stable. The control design of switched LPV systems whose subsystems are all unstable has not been properly investigated. This motivates the present study.

In addition, the tracking performance is an important requirement in practical control systems and has been extensively applied in aero-engines control, signal processing, and other fields [19–21]. The main objective of output tracking control is to force the output of a plant to track a desired reference signal under effective control as close as possible. The issue of output tracking control has been fairly addressed for switched systems and LPV systems [22, 23]. In [22], the robust H_∞ output tracking control problem is studied for switched systems under asynchronous switching. Reference [23] presents a non-fragile output tracking control scheme for an LPV system of flexible hypersonic air-breathing vehicles. However, to the best of our knowledge, the synthesis issue of H_∞ output tracking control for switched LPV systems has not been well investigated, owing to its difficulty in extending the extant results of switched systems to switched LPV systems. This also motivates the present study.

Turning to the aero-engines' control field, aero-engines are very complex nonlinear systems, and it is difficult to obtain an accurate model for an aero-engine because of the existence of their high complexity in the system dynamics. Nevertheless, most of control design rely on models in practice. In many cases, an aero-engine is often required to achieve very stringent performance objectives [24, 25]. Over the past few decades, there have been considerable efforts devoted to modeling and control of aero-engines [26–28]. High performance controllers have been designed by numerous researchers using various linearized approaches [29]. However, a linearized system model is an LTI system, which is usually effective in a small range of operating conditions. In recent years, many scholars have represented such systems by LPV systems [30, 31]. LPV modeling and control design methodologies for aero-engines have been developed because of the merits and advantages of LPV systems in coping with dramatic parameter variations and a large flight range [32]. However, LPV systems do not usually exhibit the desired levels of reliability and flexibility with a large parameter variation range. Therefore, it is necessary to search a more effective model to describe aero-engines under a large parameter variation range. One reasonable approach is to establish switched LPV models and design a family of switched LPV controllers.

Motivated by the aforementioned discussions, this paper focuses on the problem of H_∞ output tracking control for a class of switched LPV systems via multiple parameter-dependent Lyapunov functions. First, the tracking performance analysis and control synthesis conditions are established in terms of linear matrix inequalities (LMIs). Then, simulation results about a fan speed increment tracking problem for a switched LPV system of the aero-engine are provided to show the effectiveness of the proposed design methods. Compared with the existing works, the main contributions of the present study can be summarized as follows. (i) Even though the H_∞ output tracking control problem for each subsystem might be unsolvable, the problem for the switched LPV systems is still solved by the designed controllers and the designed switching law. (ii) The designed switching law is parameter and mode dependent, which is more reasonable than parameter-dependent or mode-dependent switching law for switched LPV systems. (iii) We present a control design scheme to solve the H_∞ speed adjustment problem of the nonlinear model of an aero-engine, and simulation result shows the effectiveness of the proposed control design method.

Notation. The notation in this paper is fairly standard. $A > 0$ ($A < 0$) represents a positive-definite (negative-definite) matrix A, and A^T denotes the transpose of a matrix A. The notation $A \geqslant 0$ ($A \leqslant 0$) means that the matrix A is positive semi-definite (respectively, negative semi-definite).

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$\|\cdot\|$ stands for the Euclidean norm, and $\bigcup$ represents the union operator of sets. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices. $\mathbb{R}_+(\mathbb{R}_-)$ means the non-negative (non-positive) real numbers. $L_2^n[0,+\infty)$ is the space of n-dimensional square integrable function vector over $[0,+\infty)$.

2. PROBLEM FORMULATION AND PRELIMINARIES

The situation of nonlinear approximation in system component can lead to a general switched LPV model. Therefore, the switched LPV system with the following form is considered:

$$\begin{aligned} \dot{x}(t) &= A_\sigma(\rho)x(t) + B_\sigma(\rho)u(t) + E_\sigma(\rho)\omega(t), \\ y(t) &= C_\sigma(\rho)x(t) + D_\sigma(\rho)u(t) + F_\sigma(\rho)\omega(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state and $u(t) \in \mathbb{R}^m$ is the control input. $\omega(t) \in L_2^n[0,+\infty)$ is an arbitrary external disturbance, $y(t) \in \mathbb{R}^q$ is the controlled output, and $\rho \in \mathbb{R}^s$ is a scheduling variable. σ: $\mathbb{R}_+ = [0,\infty) \to M = \{1,2,\dots,m\}$ is the switching signal, which is assumed to be a piecewise continuous (from the right) function depending on time or state, or both, or other variables. Here, the standard notations from [33] are adopted. The switching signal σ can be characterized by a switching sequence

$$\Sigma = \{x_0, \rho_0; (i_0,t_0), (i_1,t_1), \dots, (i_n,t_n), \dots | i_n \in M, n \in N\}, \tag{2}$$

in which t_0 is the initial time, x_0 is the initial state, ρ_0 is the initial parameter, and N is the set of nonnegative integers. When $t \in [t_k, t_{k+1})$, $\sigma(t) = i_k$, that is, the i_kth subsystem is active. Therefore, the trajectory $x(t)$ of the switched LPV system (1) is the trajectory of the i_kth subsystem when $t \in [t_k, t_{k+1})$. m is the number of models (called subsystems) of the switched LPV system. All of the state space data are continuous functions of the scheduling variable ρ. It is assumed that ρ is in a compact set $\mathcal{P} \in \mathbb{R}^s$ with its parameter variation rate bounded by $\underline{v}_k \leqslant \dot{\rho}_k \leqslant \bar{v}_k$ for $k = 1,\dots,s$. The parameter value is measurable in real time.

Remark 1
For a general nonlinear system

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t), \omega(t)), \\ y(t) &= h(x(t), u(t), \omega(t)), \end{aligned} \tag{3}$$

it is often difficult to design nonlinear control scheme. In order to study the nonlinear system with aid of LTI system techniques, it is generally nontrivial to use the classical linearization around different operating points derived at a global approximation by the current popular switched linear system:

$$\begin{aligned} \dot{x}(t) &= A_\sigma x(t) + B_\sigma u(t) + E_\sigma\omega(t), \\ y(t) &= C_\sigma x(t) + D_\sigma u(t) + F_\sigma\omega(t). \end{aligned} \tag{4}$$

However, a switched linear system with LTI subsystems can hardly guarantee the global performance of the nonlinear system. In order to tackle such a problem, the LTI subsystems are replaced with the LPV subsystems, which is a sound approximation of the nonlinear system. Then, a switched LPV model is derived, which represents the nonlinear system more accurately.

The reference signal $y_r(t)$ is given by the following:

$$y_r(t) = r(t) \tag{5}$$

with $r(t) \in \mathbb{R}^q$. Then, the tracking error is as follows:

$$e(t) = y(t) - y_r(t). \tag{6}$$

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Remark 2
In practice, the reference signal is often parameter independent. Furthermore, we can easily extend the proposed results to the case of the parameter-dependent reference signal. In this case, $y_r(t) = r(t)$ is replaced by $y_r(t) = C_{r\sigma}(\rho)r(t)$. Without loss of generality, the parameter-independent reference signal $y_r(t) = r(t)$ is considered.

The purpose of this paper is to design both controllers and a switching law to enforce the output $y(t)$ of the switched LPV system (1) to track the reference signal $y_r(t)$ and satisfy an H_∞ output tracking performance. To formulate the problem more precisely, the H_∞ output tracking controller of the form

$$\begin{aligned} z(t) &= \int_{t_0}^{t} e(s)ds \\ u(t) &= u_s(t) + u_t(t) \end{aligned} \tag{7}$$

is consider. The construction of the H_∞ output tracking controller consists of two parts, the state feedback part $u_s(t) = K_{si}(\rho)x(t)$ and the tracking error integral part $u_t(t) = K_{ti}(\rho)z(t)$. To implement the controller (7), we require the knowledge of $K_{si}(\rho)$ and $K_{ti}(\rho)$, $\forall i \in M$.

Remark 3
The state feedback part $u_s(t)$, if used alone, can stabilize the switched LPV system (1) and also provide a solid foundation in tracking process. The integral term of the tracking error $u_t(t)$ is dynamic compensation through the introduction of the vector $z(t)$. This is indispensable because the state feedback part $u_s(t)$ cannot guarantee an H_∞ output tracking performance without the compensation $u_t(t)$. Therefore, the integral term of the tracking error $u_t(t)$ can effectively eliminate the steady-state tracking error.

Combining (1), (6), and (7), we have the following:

$$\begin{aligned} \dot{z}(t) &= e(t) = y(t) - y_r(t) \\ &= C_\sigma(\rho)x(t) + D_\sigma(\rho)u(t) + F_\sigma(\rho)\omega(t) - r(t) \\ &= (C_\sigma(\rho) + D_\sigma(\rho)K_{si}(\rho))\, x(t) + D_\sigma(\rho)K_{ti}(\rho)z(t) + F_\sigma(\rho)\omega(t) - r(t). \end{aligned} \tag{8}$$

Augmenting system (1) with (8), we have the augmented system:

$$\begin{aligned} \dot{\eta}(t) &= \left(\bar{A}_\sigma(\rho) + \bar{B}_\sigma(\rho)\mathbb{K}_\sigma(\rho)\right)\eta(t) + \bar{E}_\sigma(\rho)\bar{\omega}(t), \\ e(t) &= \left(\bar{C}_\sigma(\rho) + D_\sigma(\rho)\mathbb{K}_\sigma(\rho)\right)\eta(t) + \bar{F}_\sigma(\rho)\bar{\omega}(t), \end{aligned} \tag{9}$$

where

$$\begin{aligned} &\eta(t) = \begin{bmatrix} x^T(t) & z^T(t) \end{bmatrix}^T, \ \bar{\omega}(t) = \begin{bmatrix} \omega^T(t) & r^T(t) \end{bmatrix}^T, \ \mathbb{K}_i(\rho) = \begin{bmatrix} K_{si}(\rho) & K_{ti}(\rho) \end{bmatrix}, \\ &\bar{A}_i(\rho) = \begin{bmatrix} A_i(\rho) & 0 \\ C_i(\rho) & 0 \end{bmatrix}, \ \bar{B}_i(\rho) = \begin{bmatrix} B_i(\rho) \\ D_i(\rho) \end{bmatrix}, \ \bar{E}_i(\rho) = \begin{bmatrix} E_i(\rho) & 0 \\ F_i(\rho) & -I \end{bmatrix}, \\ &\bar{C}_i(\rho) = \begin{bmatrix} C_i(\rho) & 0 \end{bmatrix}, \ \bar{F}_i(\rho) = \begin{bmatrix} F_i(\rho) & -I \end{bmatrix}. \end{aligned}$$

We now introduce some assumptions and definitions, which are useful in our later development.

Assumption 1
For all $\rho \in \mathcal{P}$, $(A_i(\rho), B_i(\rho), \text{and} C_i(\rho))$ triple is stabilizable and detectable.

Assumption 2
Only a finite number of switchings happen in any finite time interval.

Definition 1
Internal stability. The augmented system (9) is said to be internally stable, if the augmented system (9) with $\bar{\omega}(t) = 0$ is asymptotically stable under the control law (7) and a switching law σ.

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Then, the H_∞ output tracking control problem for the switched LPV system (1) can be stated as follows.

Definition 2
The switched LPV system (1) is said to have an H_∞ output tracking performance, if there exists a tracking controller in the form of (7) and a switching law σ such that the following conditions are satisfied:

1. *Internal stability.* The augmented system (9) is internally stable.
2. *H_∞ output tracking performance.* Under zero initial condition, for all nonzero $\bar{\omega}(t) \in L_2^n[0, +\infty)$, the tracking error $e(t)$ satisfies the following:

$$\int_0^\infty e^T(t)e(t)dt \leqslant \gamma^2 \int_0^\infty \bar{\omega}^T(t)\bar{\omega}(t)dt. \tag{10}$$

3. MAIN RESULTS

The objective of this section is to design the H_∞ output tracking controller (7) and a parameter and mode-dependent switching law $\sigma(\eta(t), \rho(t))$ such that the switched LPV system (1) has an H_∞ output tracking performance. First, a parameter-dependent state space partition method is presented, and a parameter and mode-dependent switching law is designed. Then, we give the sufficient condition ensuring the H_∞ output tracking performance for the switched LPV system (1). Finally, the H_∞ output tracking control gain is designed.

3.1. Switching law design

In this subsection, we show how to design a parameter and mode-dependent switching law $\sigma(\eta(t), \rho(t))$ to achieve the H_∞ output tracking performance with the help of the parameter-dependent H_∞ output tracking controller given in (7).

First of all, suppose that we have continuous functions $V_i(\eta, \rho), \mu_i(\eta, \rho),\ i = 1, 2, \ldots, m$, satisfying $\mu_i(\eta, \rho) \leqslant 0, \forall i$. Let

$$\Omega_i = \left\{(\eta, \rho) | V_i(\eta, \rho) - V_j(\eta, \rho) + \mu_i(\eta, \rho) \leqslant 0, j = 1, 2, \ldots, m\right\}, \tag{11}$$

$$\tilde{\Omega}_{ij} = \left\{(\eta, \rho) | V_i(\eta, \rho) - V_j(\eta, \rho) + \mu_i(\eta, \rho) = 0\right\}, i \neq j. \tag{12}$$

Then, the set $\tilde{\Omega}_i = \bigcup_{j=1, j\neq i}^m \tilde{\Omega}_{ij}$ contains the boundary of Ω_i. We now show $\bigcup_{i=1}^m \Omega_i = \mathbb{R}^{n+q} \times \mathbb{R}^s$. In fact, if this is false, namely, there exists $(\eta, \rho) \in \mathbb{R}^{n+q} \times \mathbb{R}^s$ satisfying $(\eta, \rho) \notin \Omega_i, \forall i$, then we have an integer q and a sequence $i_1, \ldots, i_q, i_k \neq i_{k+1}, k = 1, 2, \ldots, q$ with i_{q+1} being considered as i_1, such that

$$V_{i_k}(\eta, \rho) - V_{i_{k+1}}(\eta, \rho) + \mu_{i_k}(\eta, \rho) > 0. \tag{13}$$

For the inequality (13), taking the sum over k yields the following:

$$\sum_{k=1}^q \left(V_{i_k}(\eta, \rho) - V_{i_{k+1}}(\eta, \rho) + \mu_{i_k}(\eta, \rho)\right) = \sum_{k=1}^q \mu_{i_k}(\eta, \rho) > 0,$$

which contradicts $\mu_i(\eta, \rho) \leqslant 0, \forall i$.

The set Ω_i has the property that if $(\eta, \rho) \in \Omega_i \bigcap \tilde{\Omega}_{ij}$ for some i, j and $(\eta, \rho) \in \mathbb{R}^{n+q} \times \mathbb{R}^s$, then, $(\eta, \rho) \in \Omega_j$. In fact, $(\eta, \rho) \in \Omega_i \bigcap \tilde{\Omega}_{ij}$ means that $V_i(\eta, \rho) - V_k(\eta, \rho) + \mu_i(\eta, \rho) \leqslant 0$ for any k and $V_i(\eta, \rho) - V_j(\eta, \rho) + \mu_i(\eta, \rho) = 0$. Thus, $V_j(\eta, \rho) = V_i(\eta, \rho) + \mu_i(\eta, \rho)$. This in turn gives the following:

$$V_j(\eta, \rho) - V_k(\eta, \rho) + \mu_j(\eta, \rho) = V_i(\eta, \rho) - V_k(\eta, \rho) + \mu_i(\eta, \rho) + \mu_j(\eta, \rho) \leqslant 0.$$

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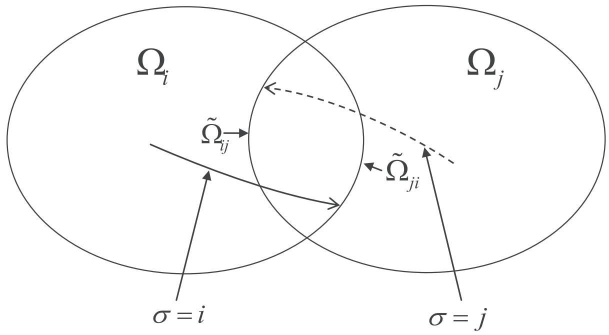

Figure 1. The parameter and mode-dependent switching law.

Now, we design the parameter and mode-dependent switching law as follows:

$$\begin{aligned} \sigma(t) &= i, \quad \text{if } \sigma(t^-) = i \text{ and } (\eta(t), \rho(t)) \in \text{ int } \Omega_i, \\ \sigma(t) &= j, \quad \text{if } \sigma(t^-) = i \text{ and } (\eta(t), \rho(t)) \in \tilde{\Omega}_{ij}. \end{aligned} \tag{14}$$

According to the switching law (14), let $\sigma(0) = i$, if $(\eta(0), \rho(0))$ belongs to Ω_i alone. Let $\sigma(0) = \arg \min_{(\eta(0),\rho(0)) \in \Omega_i,\, i=1,2,\cdots,m} \{V_i(\eta(0), \rho(0))\}$, if $(\eta(0), \rho(0))$ belongs to the intersection of some overlapping regions. The switching law (14) is shown in Figure 1.

Remark 4
Any set Ω_i with empty interior is excluded from the construction of the switching law (14). If $\sigma(t^-) = i$ and $(\eta(t), \rho(t)) \in$ int Ω_i, then the trajectory will remain in Ω_i until it hits the boundary $\tilde{\Omega}_{ij}$. Thus, once the trajectory enters Ω_i, it will remain in Ω_i until it hits the boundary in $\tilde{\Omega}_{ij}$ and then enters Ω_j. In other words, the switching from the ith subsystem to the jth subsystem only occurs on $\tilde{\Omega}_{ij}$.

Remark 5
Unlike parameter-free switched systems, the behavior of the switched LPV system (1) is largely determined by both the submodes and parameter. Therefore, it is natural and necessary to take both submodes and parameter into account when designing a switching law. Based on this consideration, we design the switching law (14), which is parameter and mode dependent. When the set $\Omega_i, i \in M$ is mode dependent but parameter independent, the switching law given by (14) degenerates exactly into the 'hysteresis-type min-switching' strategy [10].

3.2. *The H_∞ output tracking controller design*

To solve the H_∞ output tracking control problem for the switched LPV system (1), we give the following lemma.

Lemma 1
Consider the augmented system (9). If there exist matrix functions $P_i(\rho), Q_i(\rho) : \mathbb{R}^s \to \mathbb{R}^{(n+q)\times(n+q)}$ satisfying $P_i(\rho) > 0, Q_i(\rho) \leqslant 0$ and continuous functions $\beta_{ij}(\rho) : \mathbb{R}^s \to \mathbb{R}_-$ satisfying $\beta_{ij}(\rho) \leqslant 0$, such that for any $\rho \in \mathcal{P}$ and $i, j \in M$,

$$P_i(\rho)\tilde{A}_i(\rho) + \tilde{A}_i^T(\rho)P_i(\rho) + \dot{\rho}\frac{\partial P_i(\rho)}{\partial \rho} + \sum_{j=1}^{m} \beta_{ij}(\rho)\left(P_i(\rho) - P_j(\rho) + Q_i(\rho)\right) < 0, \tag{15}$$

where

$$\tilde{A}_i(\rho) = \bar{A}_i(\rho) + \bar{B}_i(\rho)\mathbb{K}_i(\rho),$$

then, the augmented system (9) is internally stable under the controller (7) and the switching law (14).

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Proof
For the augmented system (9) with $\bar{\omega}(t) \equiv 0$, choose the multiple parameter-dependent Lyapunov functions as follows:

$$V_i(\eta,\rho) = \eta^T(t)P_i(\rho)\eta(t). \tag{16}$$

Computing the time derivative alone of the state trajectory of the system (9), we have the following:

$$\begin{aligned}\dot{V}_i(\eta,\rho) &= \dot{\eta}^T(t)P_i(\rho)\eta(t) + \eta^T(t)P_i(\rho)\dot{\eta}(t) + \eta^T(t)\dot{P}_i(\rho)\eta(t)\\ &= \eta^T(t)\left(P_i(\rho)\left(\bar{A}_i(\rho)+\bar{B}_i(\rho)\mathbb{K}_i(\rho)\right)+\left(\bar{A}_i(\rho)+\bar{B}_i(\rho)\mathbb{K}_i(\rho)\right)^T P_i(\rho)+\dot{\rho}\frac{\partial P_i(\rho)}{\partial\rho}\right)\eta(t)\\ &= \eta^T(t)\left(P_i(\rho)\tilde{A}_i(\rho) + \tilde{A}_i^T(\rho)P_i(\rho) + \dot{\rho}\frac{\partial P_i(\rho)}{\partial\rho}\right)\eta(t).\end{aligned} \tag{17}$$

Pre-multiplying and post-multiplying the inequality (15) by $\eta^T(t)$ and $\eta(t)$, respectively, we obtain the following:

$$\dot{V}_i(\eta,\rho) + \sum_{j=1}^{m}\beta_{ij}(\rho)\left(V_i(\eta,\rho) - V_j(\eta,\rho) + \mu_i(\eta,\rho)\right) < 0, \quad \eta \neq 0, \tag{18}$$

where $\mu_i(\eta,\rho) = \eta^T(t)Q_i(\rho)\eta(t)$. Because $\beta_{ij}(\rho) \leqslant 0$ and $V_i(\eta,\rho) - V_j(\eta,\rho) + \mu_i(\eta,\rho) \leqslant 0$, $(\eta,\rho) \in \Omega_i, i \in M$, the inequality (18) tells us that $\dot{V}_i(\eta,\rho) < 0, (\eta,\rho) \in \Omega_i, i \in M$ under the switching law (14).

For $\forall i, j$, according to the switching law (14), at each switching time, we have the following:

$$V_i(\eta,\rho) - V_j(\eta,\rho) + \mu_i(\eta,\rho) = 0, \tag{19}$$

which implies that $V_j(\eta,\rho) \leqslant V_i(\eta,\rho)$. Therefore, the augmented system (9) with $\bar{\omega}(t) \equiv 0$ is asymptoticly stable under the controller (7) and the switching law (14), which further implies that the augmented system (9) is internally stable. This completes the proof. □

When all the subsystems of the switched LPV system are stable, a systematic switching LPV control design method for the switched LPV systems is presented in [16], and the H_∞ control for switched LPV systems is handled in [18]. However, many switched LPV systems in practice may have unstable subsystems. Thus, a natural question arises: When each subsystem of the switched LPV system is not stable and the H_∞ output tracking control problem for each subsystem might be unsolvable, can we still solve the problem for the switched LPV system by dual design of controllers and a switching law? Theorem 1 gives an answer to the question.

Theorem 1
Consider the augmented system (9). If there exist matrix functions $P_i(\rho), Q_i(\rho) : \mathbb{R}^s \to \mathbb{R}^{(n+q)\times(n+q)}$ satisfying $P_i(\rho) > 0, Q_i(\rho) \leqslant 0$ and continuous functions $\beta_{ij}(\rho) : \mathbb{R}^s \to \mathbb{R}_-$ satisfying $\beta_{ij}(\rho) \leqslant 0$, for $\forall\rho \in \mathcal{P}$ and $\forall i, j \in M$, such that the inequalities

$$\begin{bmatrix} \Lambda_i(\rho) & P_i(\rho)\bar{E}_i(\rho) & \left(\bar{C}_i(\rho) + \bar{D}_i(\rho)\mathbb{K}_i(\rho)\right)^T \\ * & -\gamma_i I & \bar{F}_i^T(\rho) \\ * & * & -\gamma_i I \end{bmatrix} < 0 \tag{20}$$

hold, where

$$\begin{aligned}\Lambda_i(\rho) &= P_i(\rho)\left(\bar{A}_i(\rho) + \bar{B}_i(\rho)\mathbb{K}_i(\rho)\right) + \left(\bar{A}_i(\rho) + \bar{B}_i(\rho)\mathbb{K}_i(\rho)\right)^T P_i(\rho) + \dot{\rho}\frac{\partial P_i(\rho)}{\partial\rho}\\ &\quad + \sum_{j=1}^{m}\beta_{ij}(\rho)\left(P_i(\rho) - P_j(\rho) + Q_i(\rho)\right),\end{aligned}$$

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then, under the switching law (14), the controller (7) solves the H_∞ output tracking control problem for the switched LPV system (1), and its H_∞ performance $\int_0^\infty e^T(t)e(t)dt \leqslant \gamma^2 \int_0^\infty \bar{\omega}^T(t)\bar{\omega}(t)dt$ is achieved with $\gamma = \max_{\forall i \in M} \gamma_i$.

Proof

It is easily seen from (20) that the augmented system (9) is internally stable. Choosing the multiple parameter-dependent Lyapunov functions as (16), we have the following:

$$\begin{aligned}\dot{V}_i(\eta,\rho) &= \dot{\eta}^T(t)P_i(\rho)\eta(t) + \eta^T(t)P_i(\rho)\dot{\eta}(t) + \eta^T(t)\dot{P}_i(\rho)\eta(t)\\ &= \eta^T(t)\left(P_i(\rho)\left(\bar{A}_i(\rho)+\bar{B}_i(\rho)\mathbb{K}_i(\rho)\right)+\left(\bar{A}_i(\rho)+\bar{B}_i(\rho)\mathbb{K}_i(\rho)\right)^T P_i(\rho) + \dot{\rho}\frac{\partial P_i(\rho)}{\partial \rho}\right)\eta(t)\\ &\quad + \bar{\omega}^T(t)\bar{E}_i^T(\rho)P_i(\rho)\eta(t) + \eta^T(t)P_i(\rho)\bar{E}_i(\rho)\bar{\omega}(t).\end{aligned} \tag{21}$$

By Schur complement lemma, the condition (20) is equivalent to the following condition:

$$\begin{aligned}&\begin{bmatrix}\Lambda_i(\rho) & P_i(\rho)\bar{E}_i(\rho)\\ * & -\gamma_i I\end{bmatrix} + \frac{1}{\gamma_i}\begin{bmatrix}\tilde{C}_i^T(\rho)\\ \bar{F}_i^T(\rho)\end{bmatrix}\begin{bmatrix}\tilde{C}_i(\rho) & \bar{F}_i(\rho)\end{bmatrix}\\ &= \begin{bmatrix}\Lambda_i(\rho) + \frac{1}{\gamma_i}\tilde{C}_i^T(\rho)\tilde{C}_i(\rho) & P_i(\rho)\bar{E}_i(\rho) + \frac{1}{\gamma_i}\tilde{C}_i^T(\rho)\bar{F}_i(\rho)\\ * & -\gamma_i I + \frac{1}{\gamma_i}\bar{F}_i^T(\rho)\bar{F}_i(\rho)\end{bmatrix}\\ &< 0,\end{aligned} \tag{22}$$

where

$$\tilde{C}_i(\rho) = \bar{C}_i(\rho) + \bar{D}_i(\rho)\mathbb{K}_i(\rho).$$

Further, pre-multiplying and post-multiplying the inequality (22) by $\begin{bmatrix}\eta^T(t) & \bar{\omega}^T(t)\end{bmatrix}$ and $[\eta(t) \quad \bar{\omega}(t)]^T$ yields the following:

$$\begin{aligned}&\dot{V}_i(\eta,\rho) + \sum_{j=1}^{m}\beta_{ij}(\rho)\left(V_i(\eta,\rho) - V_j(\eta,\rho) + \mu_i(\eta,\rho)\right)\\ &+ \frac{1}{\gamma_i}e^T(t)e(t) - \gamma_i\bar{\omega}^T(t)\bar{\omega}(t) < 0, \quad \eta \neq 0,\end{aligned} \tag{23}$$

where $\mu_i(\eta,\rho) = \eta^T(t)Q_i(\rho)\eta(t)$. Because $\beta_{ij}(\rho) \leqslant 0$, the inequality (23) implies the following:

$$\dot{V}_i(\eta,\rho) + \frac{1}{\gamma_i}e^T(t)e(t) - \gamma_i\bar{\omega}^T(t)\bar{\omega}(t) < 0 \tag{24}$$

in each subset Ω_i, $i \in M$.

According to the switching law (14), at each switching time, we have the following:

$$V_i(\eta,\rho) - V_j(\eta,\rho) + \mu_i(\eta,\rho) = 0.$$

Integrating both sides of the inequality (24) under zero initial condition, we get the following:

$$V(\eta(t),\rho(t)) - V(\eta(0),\rho(0)) + \int_0^t\left(\frac{1}{\gamma_i}e^T(t)e(t) - \gamma_i\bar{\omega}^T(t)\bar{\omega}(t)\right)dt \leqslant 0. \tag{25}$$

Due to $V(\eta(t),\rho(t)) \geqslant 0$ and $V(\eta(0),\rho(0)) = 0$, we have $\int_0^\infty e^T(t)e(t)dt \leqslant \gamma^2 \int_0^\infty \bar{\omega}^T(t)\bar{\omega}(t)dt$ with $\gamma = \max_{\forall i \in M}\gamma_i$.

Therefore, under the switching law (14), the controller (7) solves the H_∞ output tracking control problem for the switched LPV system (1). This completes the proof. □

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Remark 6
Because the terms $P_i(\rho)\bar{B}_i(\rho)\mathbb{K}_i(\rho)$ and $\mathbb{K}_i^T(\rho)\bar{B}_i^T(\rho)P_i(\rho)$ appear in condition (20), the synthesis condition for the controller (7) is generally non-convex and thus may be difficult to solve. In order to obtain the desired controller $\mathbb{K}_i(\rho)$, we give the following result.

Theorem 2
Consider the augmented system (9). If there exist matrix functions $X_i(\rho), \tilde{Q}_i(\rho) : \mathbb{R}^s \to \mathbb{R}^{(n+q)\times(n+q)}$ satisfying $X_i(\rho) > 0, \tilde{Q}_i(\rho) \leqslant 0$ and matrix functions $Y_i(\rho) : \mathbb{R}^s \to \mathbb{R}^{(n+q)\times m}$ and continuous functions $\beta_{ij}(\rho) : \mathbb{R}^s \to \mathbb{R}_-$ satisfying $\beta_{ij}(\rho) \leqslant 0$, for $\forall \rho \in \mathcal{P}$ and $\forall i, j \in M$, such that

$$\begin{bmatrix} \Xi_i(\rho) & \bar{E}_i(\rho) & X_i(\rho)\bar{C}_i^T(\rho) + Y_i^T(\rho)D_i^T(\rho) & S_i(\rho) \\ * & -\gamma_i I & \bar{F}_i^T(\rho) & 0 \\ * & * & -\gamma_i I & 0 \\ * & * & * & -M_i(\rho) \end{bmatrix} < 0 \tag{26}$$

hold, where

$$\Xi_i(\rho) = X_i(\rho)\bar{A}_i^T(\rho) + \bar{A}_i(\rho)X_i(\rho) + Y_i^T(\rho)\bar{B}_i^T(\rho) + \bar{B}_i(\rho)Y_i(\rho) - \sum_{k=1}^{s} \dot{\rho}_k \frac{\partial X_i(\rho)}{\partial \rho_k} + \sum_{j=1, i\neq j}^{m} \beta_{ij}(\rho)X_i(\rho) + \sum_{j=1}^{m} \beta_{ij}(\rho)\tilde{Q}_i(\rho),$$

$$S_i(\rho) = \left[\sqrt{-\beta_{i1}(\rho)}X_i(\rho), \ldots, \sqrt{-\beta_{ii-1}(\rho)}X_i(\rho), \sqrt{-\beta_{ii+1}(\rho)}X_i(\rho), \ldots, \sqrt{-\beta_{im}(\rho)}X_i(\rho)\right],$$

$$M_i(\rho) = diag\left\{X_1(\rho), \ldots, X_{i-1}(\rho), X_{i+1}(\rho), \ldots, X_m(\rho)\right\},$$

then, under the switching law (14), the controller (7) solves the H_∞ output tracking control problem for the switched LPV system (1), and its H_∞ output tracking performance $\int_0^\infty e^T(t)e(t)dt \leqslant \gamma^2 \int_0^\infty \bar{\omega}^T(t)\bar{\omega}(t)dt$ is achieved with $\gamma = \max_{\forall i \in M} \gamma_i$. Moreover, the controller gains are given by $\mathbb{K}_i(\rho) = Y_i(\rho)X_i^{-1}(\rho), \ \ \forall i \in M$.

Proof
It is easy to see that the H_∞ output tracking control problem for the switched LPV system (1) is solvable if the conditions (20) are satisfied.

Then, pre-multiplying and post-multiplying the inequality (20) by diag $\{P_i^{-1}(\rho) \ \ I \ \ I\}$ and performing a congruence transformation to (20) by $X_i(\rho) = P_i^{-1}(\rho), Y_i(\rho) = \mathbb{K}_i(\rho)X_i(\rho)$, we have the following:

$$\begin{bmatrix} \bar{\Xi}_i(\rho) & \bar{E}_i(\rho) & X_i(\rho)\bar{C}_i^T(\rho) + Y_i^T(\rho)D_i^T(\rho) \\ * & -\gamma_i I & \bar{F}_i^T(\rho) \\ * & * & -\gamma_i I \end{bmatrix} < 0, \tag{27}$$

where

$$\bar{\Xi}_i(\rho) = X_i(\rho)\bar{A}_i^T(\rho) + \bar{A}_i(\rho)X_i(\rho) + Y_i^T(\rho)\bar{B}_i^T(\rho) + \bar{B}_i(\rho)Y_i(\rho) + \dot{\rho}X_i(\rho)\frac{\partial X_i^{-1}(\rho)}{\partial \rho}X_i(\rho) + \sum_{j=1}^{m} \beta_{ij}(\rho)X_i(\rho)X_i^{-1}(\rho)X_i(\rho) - \sum_{j=1}^{m} \beta_{ij}(\rho)X_i(\rho)X_j^{-1}(\rho)X_i(\rho) + \sum_{j=1}^{m} \beta_{ij}(\rho)X_i(\rho)Q_i(\rho)X_i(\rho).$$

We now deal with the terms $\dot{\rho}X_i(\rho)\frac{\partial X_i^{-1}(\rho)}{\partial \rho}X_i(\rho)$. Because $X_i(\rho)X_i^{-1}(\rho) = I$ hold for arbitrary positive definite matrix functions, we have $\frac{\partial (X_i(\rho)X_i^{-1}(\rho))}{\partial t} = X_i(\rho)\dot{\rho}\frac{\partial X_i^{-1}(\rho)}{\partial \rho} + \dot{\rho}\frac{\partial X_i(\rho)}{\partial \rho}X_i^{-1}(\rho) = 0$. Further, $X_i(\rho)\dot{\rho}\frac{\partial X_i^{-1}(\rho)}{\partial \rho} = \dot{\rho}\frac{\partial X_i(\rho)}{\partial \rho}X_i^{-1}(\rho)$. Again, we obtain $\dot{\rho}\frac{\partial X_i^{-1}(\rho)}{\partial \rho} = -\dot{\rho}X_i^{-1}(\rho)\frac{\partial X_i(\rho)}{\partial \rho}X_i^{-1}(\rho)$, which further implies that

$$
\begin{aligned}
\bar{\Xi}_i(\rho) &= X_i(\rho)\bar{A}_i^T(\rho) + \bar{A}_i(\rho)X_i(\rho) + Y_i^T(\rho)\bar{B}_i^T(\rho) + \bar{B}_i(\rho)Y_i(\rho) - \dot{\rho}\frac{\partial X_i(\rho)}{\partial \rho} \\
&\quad + \sum_{j=1,i\neq j}^{m} \beta_{ij}(\rho)X_i(\rho) - \sum_{j=1,i\neq j}^{m} \beta_{ij}(\rho)X_i(\rho)X_j^{-1}(\rho)X_i(\rho) + \sum_{j=1}^{m}\beta_{ij}(\rho)\tilde{Q}_i(\rho) \\
&= X_i(\rho)\bar{A}_i^T(\rho) + \bar{A}_i(\rho)X_i(\rho) + Y_i^T(\rho)\bar{B}_i^T(\rho) + \bar{B}_i(\rho)Y_i(\rho) - \sum_{k=1}^{s}\dot{\rho}_k\frac{\partial X_i(\rho)}{\partial \rho_k} \\
&\quad + \sum_{j=1,i\neq j}^{m} \beta_{ij}(\rho)X_i(\rho) - \sum_{j=1,i\neq j}^{m} \beta_{ij}(\rho)X_i(\rho)X_j^{-1}(\rho)X_i(\rho) + \sum_{j=1}^{m}\beta_{ij}(\rho)\tilde{Q}_i(\rho),
\end{aligned}
$$

where $\tilde{Q}_i(\rho) = X_i(\rho)Q_i(\rho)X_i(\rho)$. The inequality (27) becomes the following:

$$
\begin{aligned}
&\begin{bmatrix} \Xi_i(\rho) & \bar{E}_i(\rho) & X_i(\rho)\bar{C}_i^T(\rho) + Y_i^T(\rho)D_i^T(\rho) \\ * & -\gamma I & \bar{F}_i^T(\rho) \\ * & * & -\gamma I \end{bmatrix} \\
&- \begin{bmatrix} \sum_{j=1,i\neq j}^{m}\beta_{ij}(\rho)X_i(\rho)X_j^{-1}(\rho)X_i(\rho) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} < 0.
\end{aligned} \tag{28}
$$

Applying Schur complement lemma to (28) gives (26). Therefore, if LMIs (26) hold, under the switching law (14), the controller (7) solves the H_∞ output tracking control problem for the switched LPV system (1). Moreover, the tracking controller (7) gains are given by $\mathbb{K}_i(\rho) = Y_i(\rho)X_i^{-1}(\rho), \forall i \in M$. This completes the proof. □

Remark 7

It is sometimes difficult to find an effective tool to deal with the term $\dot{\rho}_k\frac{\partial X_i(\rho)}{\partial \rho_k}$. Inspired by [15], we replace the term $\sum_{k=1}^{s}\dot{\rho}_k\frac{\partial X_i(\rho)}{\partial \rho_k}$ with the term $\sum_{k=1}^{s}\{\underline{\upsilon}_k, \bar{\upsilon}_k\}\frac{\partial X_i(\rho)}{\partial \rho}$, which represents the combination of the derivative terms $\dot{\rho}_k\frac{\partial X_i(\rho)}{\partial \rho_k}$ when $\dot{\rho}_k$ is taken as either $\underline{\upsilon}_k$ or $\bar{\upsilon}_k$. Therefore, each inequality contains 2^s coupled different LMIs. In general, we need to solve all the LMIs simultaneously.

4. EXAMPLE

In this section, we apply the proposed H_∞ output tracking control technique to a speed adjustment problem of the $GE-90$ aero-engine. In order to design the controller using a switching LPV method, the switched LPV model of the $GE - 90$ aero-engine is firstly established via the linearization technique at multiple operating points and the system identification technique. Then, we implement the proposed control design scheme to a nonlinear model of the $GE - 90$ aero-engine to test the effectiveness of the developed result.

4.1. Derivation of the switched linear parameter-varying model

Aero-engines are highly complex nonlinear systems, and it is difficult to obtain a complete mathematical description. In the existing literature, two classical LPV models of aero-engines have been

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validated with the help of the gain scheduling method. The experimental results have also confirmed that the two LPV models are feasible for the control design of aero-engines. However, it can also be found that any single approximate model does not fully exhibit the desired levels of reliability and flexibility with dramatic parameter variations and a large flight range. In order to improve design accuracy of the LPV model, it is necessary to choose a more accurate model for control design in real time. This is the main reason why we consider the switching of multi-models.

Therefore, the switching technique [8] and the idea of the multi-model control [34] are introduced to establish a switched LPV model of $GE-90$ aero-engine. The switched LPV model contains two LPV subsystems; one is the linearization model from cycle deck data, and the other is the system identification model from curve-fit [35]. The linearization model is a high-fidelity simulation model used for partial derivative information, because the partial derivative information based on the massive information can be obtained across a wide range of operating conditions [31, 35]. The system identification model is more systematic. Also, the measured noise and its impact on model quality are addressed explicitly [35, 36]. It is commonly known that the Lyapunov functions, to a large degree, can reflect the behavior of systems. Therefore, the mode-dependent switching law is designed in accordance with the variation of the Lyapunov functions to better achieve the control objective. From (14), it is seen that the subsystem with the minimum Lyapunov functions is active at every switching time. The switched LPV model is more satisfactory than any one of the two subsystems, and simulation results also show the effectiveness of control design based on the switched LPV model.

The data of $GE-90$ aero-engine model are borrowed from [35]. In order to design a controller, a switched LPV model of $GE-90$ aero-engine via the linearization technique and the system identification technique is given by the following:

$$\begin{aligned}\begin{bmatrix}\triangle \dot{N_f}(t)\\ \triangle \dot{N_c}(t)\end{bmatrix} &= A_\sigma(m,h)\begin{bmatrix}\triangle N_f(t)\\ \triangle N_c(t)\end{bmatrix} + B_\sigma(m,h)\triangle W_F(t) + E_\sigma(m,h)\omega(t),\\ \triangle N_f(t) &= C_\sigma(m,h)\begin{bmatrix}\triangle N_f(t)\\ \triangle N_c(t)\end{bmatrix},\end{aligned} \tag{29}$$

where $\triangle N_f = N_f - N_{f0}$ is the fan speed increment, $\triangle N_c = N_c - N_{c0}$ is the core speed increment, $\triangle W_F = W_F - W_{F0}$ is the fuel flow increment, and $y(t)$ is the output, respectively. In addition, N_f is the fan speed, N_c is the core speed, and W_F is the fuel flow. $\omega(t)$ is given by the health parameter input, and an important interpretation of a set of quantities representing engine deterioration and faults.

The parameter set $\mathcal{P} = \{(m,h)|m \in [0, 0.4], h \in [0, 1.2]\}$ is considered, in which m is the Mach number and h is the altitude normalized by 10 000. Set the bound of the parameter derivative as 0.4. $A_i(m,h)$, $B_i(m,h)$, $E_i(m,h)$, and $C_i(m,h)$, $i = 1, 2$ are parameterized in m, h by means of interpolation or fitting, which are expressed as follows:

$$A_1(m,h) = \begin{bmatrix}-3.8557 & 1.4467\\ 0.4690 & -4.7081\end{bmatrix} + m\begin{bmatrix}-0.7108 & 0.1240\\ 0.4728 & 0.2972\end{bmatrix} + h\begin{bmatrix}1.0956 & -0.3840\\ -0.1723 & 1.1545\end{bmatrix},$$

$$A_2(m,h) = \begin{bmatrix}-3.8293 & 1.4323\\ 0.4433 & -4.6533\end{bmatrix} + m\begin{bmatrix}-1.4042 & 0.3993\\ -0.4755 & -1.1028\end{bmatrix} + h\begin{bmatrix}0.8925 & -0.3218\\ 0.3193 & 0.9469\end{bmatrix},$$

$$B_1(m,h) = \begin{bmatrix}230.6739\\ 653.5547\end{bmatrix} + m\begin{bmatrix}-20.6140\\ -105.3620\end{bmatrix} + h\begin{bmatrix}14.2541\\ 61.3506\end{bmatrix},$$

$$B_2(m,h) = \begin{bmatrix}231.2195\\ 656.2097\end{bmatrix} + m\begin{bmatrix}2.3770\\ 73.2831\end{bmatrix} + h\begin{bmatrix}3.3126\\ 10.9868\end{bmatrix},$$

$$E_1(m,h) = E_2(m,h) = \begin{bmatrix}0.1\\ 0.2\end{bmatrix}, \quad C_1(m,h) = C_2(m,h) = [1 \quad 0].$$

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4.2. Controller design

Our purpose is to design the H_∞ output tracking controller in the form of (7) and the parameter and mode-dependent switching law (14) such that the fan speed increases from (2319, 8719) to (2432, 9274) according to the reference command and minimizes the disturbance effect.

The reference command supplied to $GE-90$ aero-engine is given by the following:

$$y_r(t) = r(t) = \begin{cases} -113, & 0 \leqslant t \leqslant 0.3, \\ 160t, & 0.3 \leqslant t \leqslant 1.1, \\ 0, & 1.1 \leqslant t \leqslant 20. \end{cases}$$

The disturbance is given by $\omega(t) = e^{-2t}$. Obviously, we have $\bar{\omega}(t) \in L_2[0,\infty)$. To show the effectiveness of the developed H_∞ output tracking control strategy, we conduct a comparative simulation study using the following three methods: the classical LPV control method [23, 37], the well-known average dwell-time method [15, 16], and the proposed control design method.

1: Based on the classical LPV control method, we obtain $\gamma_a = 28.4660$ and

$$X(m,h) = \begin{bmatrix} 1.9821 & 2.7118 & -6.4909 \\ 2.7118 & 19.2927 & -8.2216 \\ -6.4909 & -8.2216 & 28.4660 \end{bmatrix} + m\begin{bmatrix} -1.3167 & 0.7129 & 0.1115 \\ 0.7129 & 4.7243 & -0.6026 \\ 0.1115 & -0.6026 & 0.0000 \end{bmatrix}$$
$$+ h\begin{bmatrix} 0.6150 & -0.1928 & 0.0219 \\ -0.1928 & 1.5444 & -0.0614 \\ 0.0219 & -0.0614 & -0.0000 \end{bmatrix},$$
$$Y(m,h) = [-0.0475 \quad 0.1110 \quad -0.0579] + m[-0.0235 \quad 0.0678 \quad -0.0195]$$
$$+ h * 1.0e-003 * [0.0090 \quad -0.0312 \quad 0.0165].$$

The controller gain is given by $\mathbb{K}(m,h) = Y(m,h)X^{-1}(m,h)$.

2: Applying the well-known average dwell-time method, we have $\gamma_b = 14$, $\tau = 4.3467$, and

$$X_1(m,h) = \begin{bmatrix} 7.7928 & 2.6113 & -7.1938 \\ 3.6113 & 1.1906 & -1.3992 \\ -7.1938 & -1.3992 & 6.0888 \end{bmatrix} + m\begin{bmatrix} -7.3820 & -8.9630 & 9.8986 \\ -8.9630 & -6.9999 & 8.4685 \\ 9.8986 & 8.4685 & -3.3272 \end{bmatrix}$$
$$+ h\begin{bmatrix} 9.8001 & 5.3401 & -2.7434 \\ 5.3401 & 5.0197 & -6.0248 \\ -2.7434 & -6.0248 & 2.3849 \end{bmatrix},$$
$$X_2(m,h) = \begin{bmatrix} 6.7719 & 0.9780 & -1.9217 \\ 0.9780 & 9.8112 & -1.9543 \\ -1.9217 & -1.9543 & 1.8047 \end{bmatrix} + m\begin{bmatrix} 1.2740 & -2.6866 & -2.8254 \\ -2.6866 & 2.9260 & -2.3374 \\ -2.8254 & -2.3374 & 0.0219 \end{bmatrix}$$
$$+ h\begin{bmatrix} -4.1367 & 0.1014 & 0.7898 \\ 0.1014 & -1.2599 & 0.5689 \\ 0.7898 & 0.5689 & -0.2882 \end{bmatrix},$$
$$Y_1(m,h) = [-0.0194 \quad 0.3825 \quad -0.1589] + m[-0.1243 \quad -1.8140 \quad 0.5859]$$
$$+ h[-0.0018 \quad 0.8873 \quad -0.3304],$$
$$Y_2(m,h) = [-0.0533 \quad -0.0627 \quad -0.0124] + m[-0.7033 \quad 1.2188 \quad -0.0003]$$
$$+ h[0.1274 \quad -0.2282 \quad 0.0034].$$

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3: Based on the LMIs in Theorem 2, we obtain $\gamma_c = 10.6391$ and

$$X_1(m,h) = \begin{bmatrix} 0.4004 & 0.0871 & -0.1746 \\ 0.0871 & 0.5528 & -0.2996 \\ -0.1746 & -0.2996 & 0.9738 \end{bmatrix} + m \begin{bmatrix} 3.5131 & 0.7322 & -1.2644 \\ 0.7322 & 4.2978 & -0.9462 \\ -1.2644 & -0.9462 & 1.8307 \end{bmatrix}$$
$$+ h \begin{bmatrix} -0.8960 & -0.2470 & 0.4199 \\ -0.2470 & -1.1835 & 0.4394 \\ 0.4199 & 0.4394 & -1.0306 \end{bmatrix},$$
$$X_2(m,h) = \begin{bmatrix} 1.2010 & 0.2555 & -0.4598 \\ 0.2555 & 1.5386 & -0.5102 \\ -0.4598 & -0.5102 & 1.3809 \end{bmatrix} + m \begin{bmatrix} -3.2825 & -0.7370 & 1.2950 \\ -0.7370 & -4.0993 & 1.0578 \\ 1.2950 & 1.0578 & -2.0433 \end{bmatrix}$$
$$+ h \begin{bmatrix} 0.7435 & 0.1482 & -0.2692 \\ 0.1482 & 0.9030 & -0.1905 \\ -0.2692 & -0.1905 & 0.3099 \end{bmatrix},$$
$$Y_1(m,h) = [-0.0003 \quad 0.0023 \quad -0.0042] + m[-0.0052 \quad 0.0178 \quad -0.0002]$$
$$+ h[0.0008 \quad -0.0063 \quad 0.0023],$$

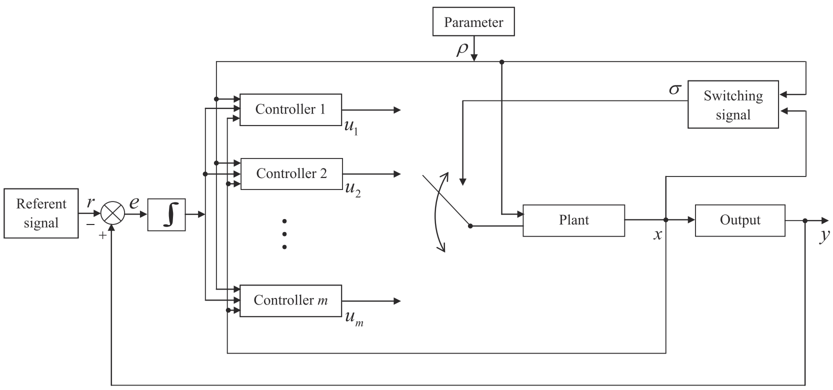

Figure 2. The architecture implementing the switched linear parameter-varying control scheme.

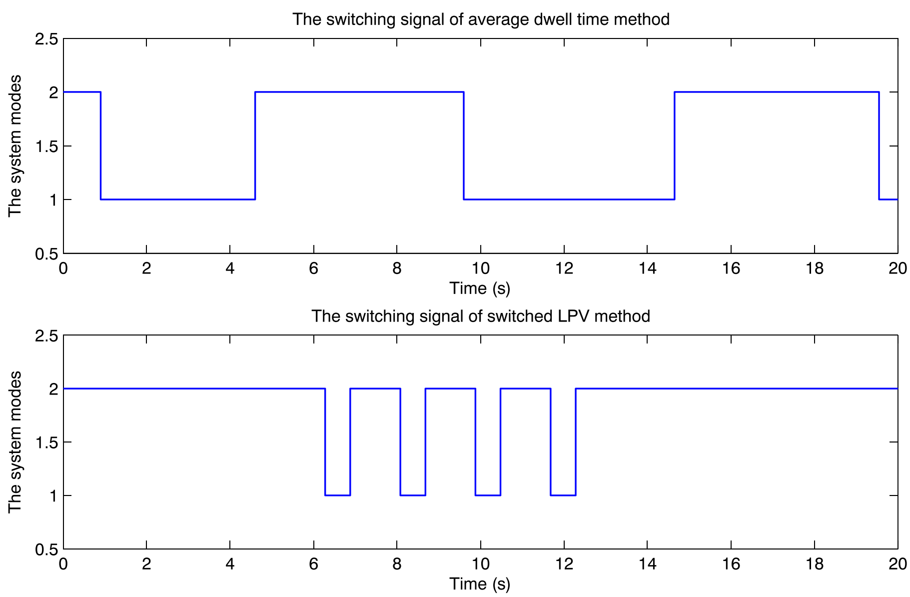

Figure 3. The switching signals.

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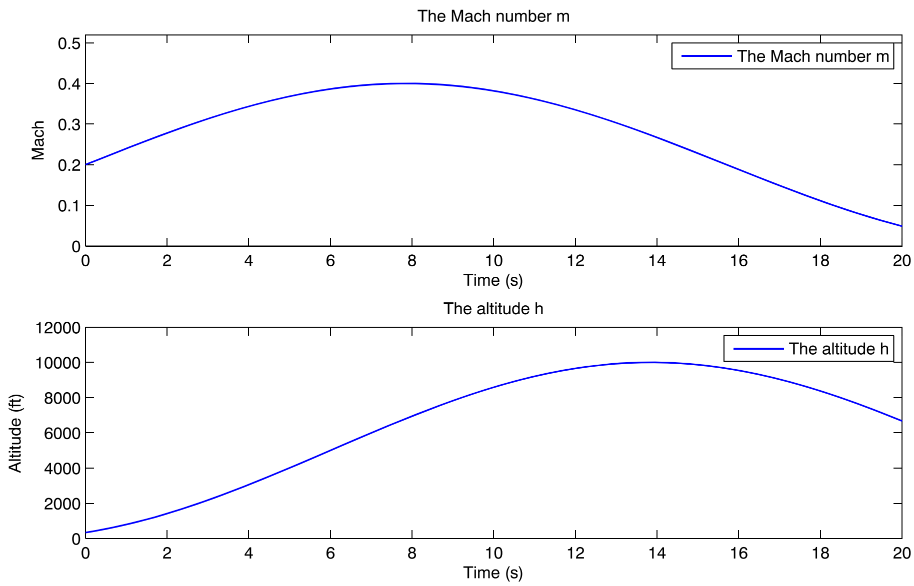

Figure 4. The trajectories of the Mach number and the altitude.

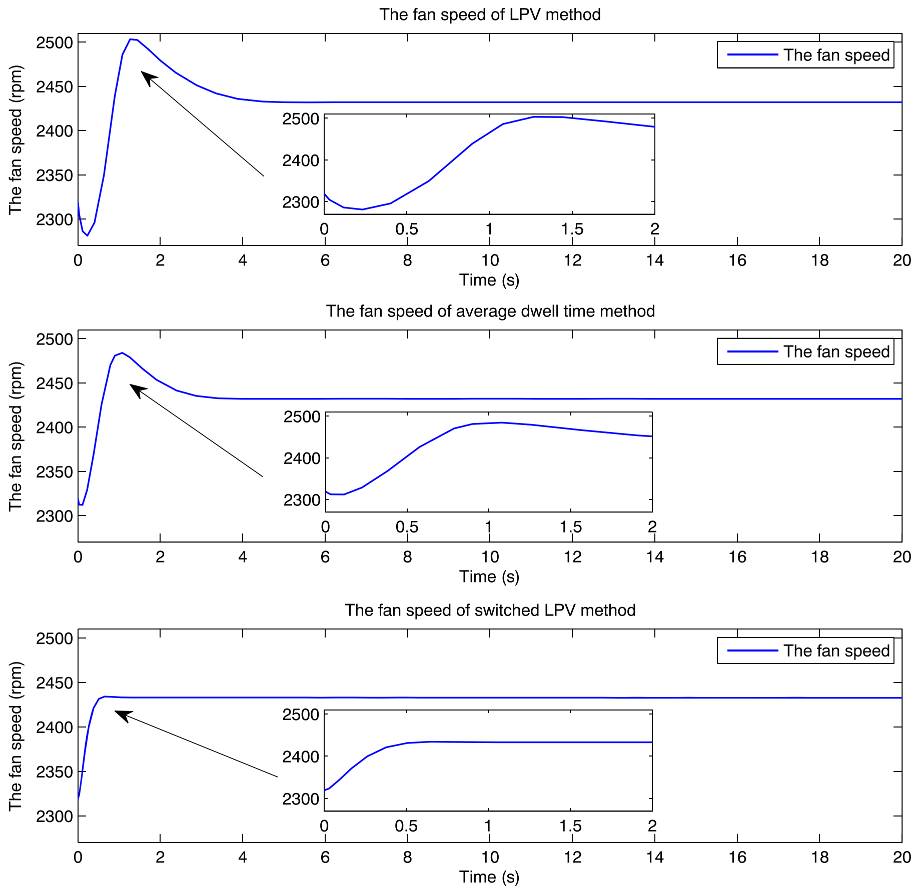

Figure 5. The fan speed. LPV, linear parameter varying.

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$$Y_2(m,h) = [-0.0015 \quad 0.0061 \quad -0.0039] + m[0.0047 \quad -0.0160 \quad 0.0007] + h[-0.0013 \quad 0.0029 \quad 0.0006].$$

The gains are given by $\mathbb{K}_i(m,h) = Y_i(m,h)X_i^{-1}(m,h)$ in 2 and 3. It becomes obvious that γ_c is the smallest H_∞ gain, so the developed design of parameter and mode-dependent switching signal minimizes the disturbance effect and exhibits better performance than the LPV control method and the average dwell-time method.

4.3. Simulation results

We then conduct the simulation and implement the proposed control design scheme to the nonlinear model of the $GE-90$ aero-engine given by [35]. The architecture implementing the switched LPV control scheme is shown in Figure 2. We also work out the following:

$$\begin{aligned} V_1(t) = {} & (2.7273 - 7.2899m + 1.7974h)\triangle N_f(t)^2 + (2.1852 - 5.9232m + 1.6747h)\triangle N_c(t)^2 \\ & + (1.2996 - 1.6398m + 1.8248h)z(t)^2 + (0.3950 + 1.1416m + 0.0322h)\triangle N_f(t)\triangle N_c(t) \\ & + (0.8564 - 1.1088m + 0.3560h)\triangle N_f(t)z(t) + (1.2736 - 3.1343m + 0.6448h)\triangle N_c(t)z(t), \end{aligned}$$

$$\begin{aligned} V_2(t) = {} & (1.0444 + 7.0375m - 1.3709h)\triangle N_f(t)^2 + (0.8086 + 5.8290m - 1.0614h)\triangle N_c(t)^2 \\ & + (0.9057 + 1.8986m - 0.0531h)z(t)^2 + (-0.1448 - 1.0764m + 0.3342h)\triangle N_f(t)\triangle N_c(t) \\ & + (0.5932 + 0.6666m - 0.0936h)\triangle N_f(t)z(t) + (0.5394 + 2.7034m - 0.5114h)\triangle N_c(t)z(t), \end{aligned}$$

where $z(t) = \int_{t_0}^{t}(\triangle N_f(s) - r(s))ds$. The switching surfaces (the boundaries equation) are given by $(3.7717 - 0.2524m + 0.4265h)\triangle N_f(t)^2 + (2.9938 - 0.0942m + 0.6133h)\triangle N_c(t)^2 + (2.2053 +$

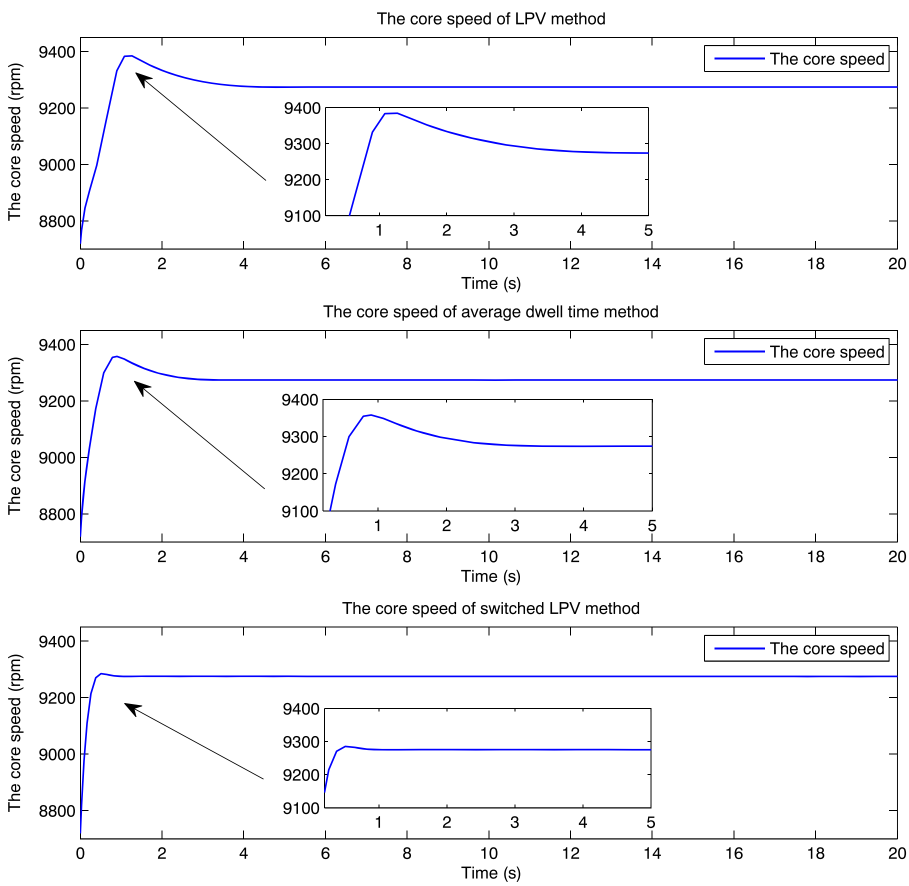

Figure 6. The core speed. LPV, linear parameter varying.

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$0.2588m+1.7717h)z(t)^2+(0.2502+0.0652m+0.3665h)\triangle N_f(t)\triangle N_c(t)+(1.4496-0.4422m+0.2624h)\triangle N_f(t)z(t)+(1.8130-0.4309m+0.1334h)\triangle N_c(t)z(t)=0$. When $t\in[0,20]$, the Lyapunov function values of the switching times are given by $V(6.28)=1.2e-3$, $V(6.88)=4e-4$, $V(8.09)=5.2e-5$, $V(8.69)=1.0e-5$, $V(9.89)=5.9e-6$, $V(10.49)=1.9e-6$, $V(11.69)=1.8e-6$, and $V(12.29)=1.1e-7$. In addition, the computational times by using the LPV control method, the average dwell-time method, and our method are 3.3445, 3.6725, and 3.4791 s, respectively. In contrast to the LPV control method, our method reduces remark-

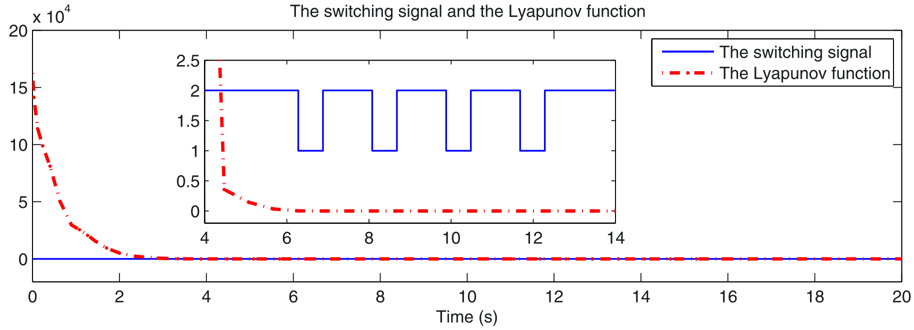

Figure 7. The Lyapunov function.

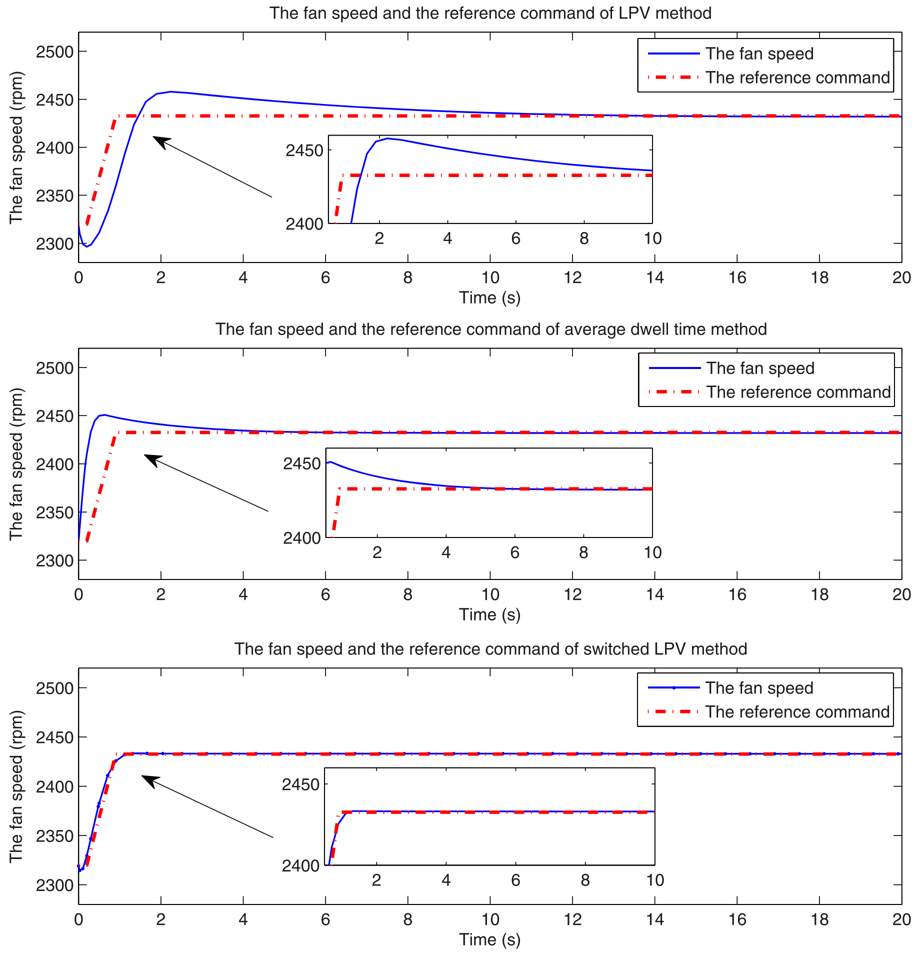

Figure 8. The fan speed and the reference command. LPV, linear parameter varying.

ably the overshoots of the speed and enhances the adjusting velocity, although the times of our method and classical LPV control method are almost the same and a little bit slow because of the switching. Compared with the average dwell-time method, the time of our method is smaller than the time of the average dwell-time method, so our method has less computation than the average dwell-time method.

The different switching signals are given in Figure 3. The time-varying Mach number and altitude profile are chosen in Figure 4. The simulation results are presented in Figures 5–9, and subplots are zoomed view of responses. Figures 5–6 describe the variations of the fan speed and the core speed, respectively. The multiple parameter-dependent Lyapunov functions are given in Figure 7. The fan

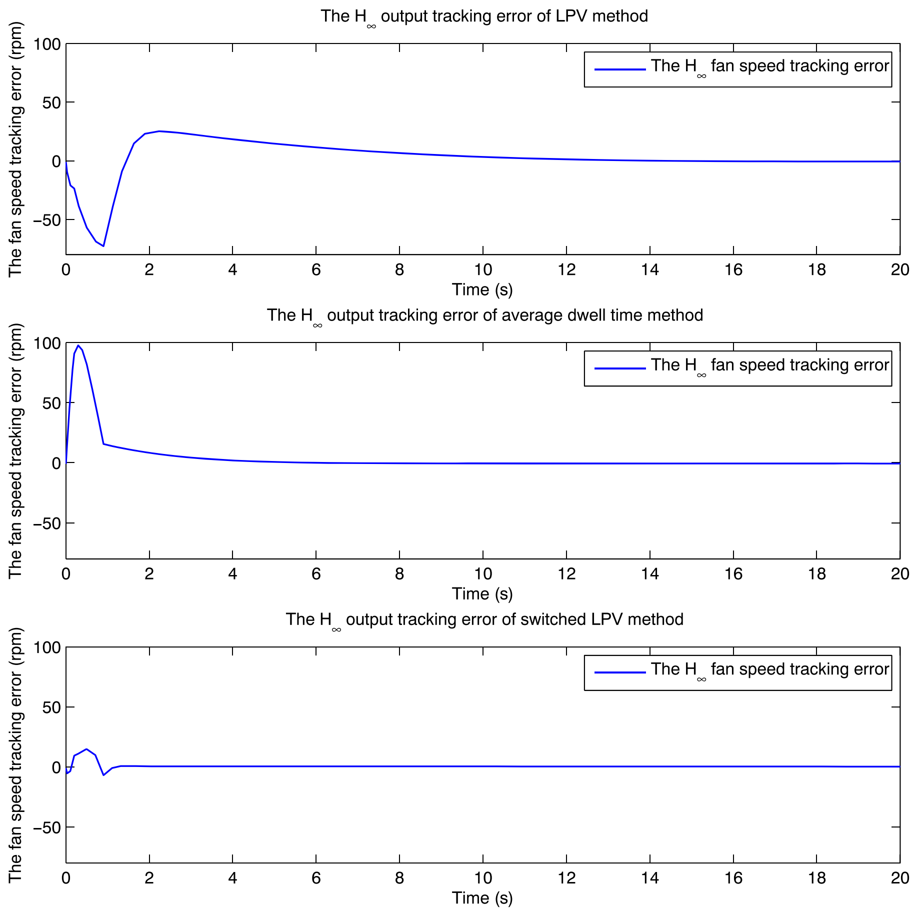

Figure 9. The H_∞ fan speed tracking error. LPV, linear parameter varying.

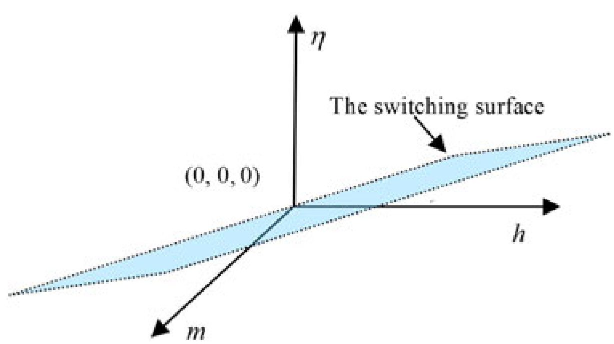

Figure 10. The sketch of the state space partition.

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speed and the reference command for the nonlinear model of the $GE - 90$ aero-engine are given in Figure 8, and the H_∞ fan speed tracking error is depicted in Figure 9. Figures 5–9 show that, compared with the LPV control method and the average dwell-time method, the proposed design of the H_∞ fan speed tracking controller significantly reduces the overshoots of the speed and enhances the adjusting velocity. Also, the transient state and the steady-state performance of the proposed design is more satisfactory because of the effect of the proposed switching LPV control scheme. The sketch of the state space partition is given in Figure 10.

5. CONCLUSIONS

In this paper, we have investigated the problem of H_∞ output tracking control for a class of switched LPV systems. A sufficient condition for H_∞ output tracking control of switched LPV systems is given in the format of LMIs, and a set of parameter and mode-dependent switching signals are designed, and a family of switching LPV controllers are developed. Even though the H_∞ output tracking control problem for each subsystem might be unsolvable, the problem for the switched LPV system is still solved by the designed controllers and the designed switching law. Finally, simulation results about an H_∞ speed adjustment problem of an aero-engine were presented to show the effectiveness of the proposed control design scheme.

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