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Modelling hierarchical decision making framework for operation of active distribution grids

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Abstract: In this study, a hierarchical decision making framework for operation of active distribution grids (ADGs) in which distribution company (Disco) and microgrids (MGs) cooperate with each other is modelled. Since there are several MGs, which may have independent decision makers, in ADGs, two levels of decision makers are considered in this framework. The mentioned framework is modelled as a bi-level optimisation problem in which Disco and MGs are considered as the leader and the followers, respectively. The resulting model is a non-linear bi-level problem which is transformed into a linear single-level problem through Karush–Kuhn–Tucker conditions and dual theory. To evaluate the effectiveness of the model, a hypothetical distribution grid is considered as the case study.

Nomenclature

Indices

j	index of MG
t	index of time
s	index of scenario
i	index of greater than or equal to zero constraint

Sets

J	set of MGs
T	set of time
S_j	set of scenarios of MG j
I	set of greater than or equal to zero constraints

Parameters

C_j^{DG}	generation cost of DG ($/MWh)
$C_{j,t}^{\mathrm{IL}}$	load curtailment contract price ($/MWh)
$E_j^{\min}$	minimum energy storage of battery (MWh)
$E_j^{\max}$	maximum energy storage of battery (MWh)
E_j^{ini}	initial energy storage of battery (MWh)
$P_j^{\mathrm{batt,\,max}}$	maximum charging/discharging power of battery (MW)
$P_{j,t,s}^{\mathrm{demand}}$	power demand (MW)
$P_j^{\mathrm{DG,\,min}}$	minimum power generation of DG (MW)
$P_j^{\mathrm{DG,\,max}}$	maximum power generation of DG (MW)
$P_j^{\mathrm{DG,\,ini}}$	initial power generation of DG (MW)
$P_{j,t}^{\mathrm{IL,\,max}}$	maximum amount of load curtailment (MW)
$P^{\max}$	maximum purchased power from wholesale electricity market (MW)
$P_{\max}^{T_j}$	maximum power exchange between Disco and MG j (MW)
$P_{j,t,s}^{\mathrm{WT}}$	power generation of wind turbine (MW)
$\mathrm{RDN}_j^{\mathrm{DG}}$	ramp down limit for DG (MW/h)
$\mathrm{RUP}_j^{\mathrm{DG}}$	ramp up limit for DG (MW/h)
ρ_t^M	wholesale electricity price ($/MWh)
$\rho^{\max}$	upper limit for price of power exchange between Disco and MGs ($/MWh)
π_s	probability of scenarios
η_j^{ch}	charging efficiency of battery
η_j^{dch}	discharging efficiency of battery

Variables

$E_{j,\,t,\,s}$	energy storage of battery (MWh)
P_t^M	purchased power from wholesale electricity market (MW)
$\hat{P}_{j,t}^D$	expected value of power exchange between Disco and MG j (MW)
$P_{j,t,s}^D$	power exchange between Disco and MG j (MW)
$P_{j,t,s}^{\mathrm{ch}}$	charging power of battery (MW)
$P_{j,t,s}^{\mathrm{dch}}$	discharging power of battery (MW)
$P_{j,t,s}^{\mathrm{DG}}$	power generation of DG (MW)
$P_{j,t,s}^{\mathrm{IL}}$	the amount of load curtailment (MW)
$U_{j,t,s}^i$	binary variable used for linearisation of the complementary slackness conditions
$\lambda_{j,t,s}^i$	dual variable ($/MWh)
ρ_t^{D}	price of power exchange between Disco and MGs ($/MWh)

Others

L^j	Lagrangian function
$C_{j,t,s}^i$	greater than or equal to zero constraint

1 Introduction

1.1 Motivation and aim

High penetration of distributed energy resources (DERs) has caused the emergence of new decision makers that changed the decision making environment in power system. To increase the operational flexibility of the system, decision makers with different and conflicting objectives should cooperate with each other. Modelling this cooperation environment, especially in distribution grid level, introduces new challenges for researchers.

In the presence of DERs, distribution systems are known as active distribution grids (ADGs) [1]. In ADGs, distribution companies (Discos) have complex operation problems in comparison with traditional distribution grids. To better managing of ADGs, their DERs should be integrated in the form of microgrids (MGs) [2]. These MGs mostly have independent decision makers that represent various characteristics based on their DERs combination (e.g. renewable energy-based distributed generators, fossil fuel-based distributed generators, energy storages, and controllable loads). This diversity makes decision making conflicts between MGs and Disco, as upper level decision maker of ADG. Managing these conflicts release inherent operational flexibility of decision makers and enhance the total productivity of ADGs. As there are two levels of decision makers, that is, Disco and the MGs, the operation problem of ADG requires a hierarchical decision making framework.

1.2 Literature review and contributions

Different aspects of MGs presence in distribution grids are investigated in the literature. Several criteria including installation and operation costs, active power losses, improve the network reliability, and environmental benefits are considered to evaluate the effects of MGs penetration on ADGs [3, 4]. Jiang and Xiaohong [5] proposed a decentralised optimal control algorithm to coordinate information and strategies among MGs. Logenthiran *et al.* [6] investigated energy resource scheduling of several MGs in isolated distribution grid using multi-agent systems. Fathi and Bevrani [7] formulated the demand management of distribution grid with connected MGs as a multi-objective optimisation problem. Energy management and trading of several MGs including demand response programmes is presented in [8]. In these studies [5–8], distribution grid is considered as coupled MGs and cooperation among them is modelled from different viewpoints.

In [9], energy management of an energy service provider including several MGs in competition with a large central production unit is investigated using bi-level optimisation problem. Kargarian *et al.* [10, 11] proposed a system of systems framework for operation problems of Disco and MGs solved using the iterative process. In each iteration, the required data is exchanged between these systems and the iterative process is continued as long as the converge conditions are satisfied. In these papers, the economic aspects of cooperation between Disco and MGs, as the main motivation of these decision makers to cooperate with each other, are not modelled properly. Moreover, the model presented in [10, 11] is considered in one time step and the dynamic behaviour of decision makers is not modelled.

In this paper, a bi-level optimisation approach is developed to model the cooperation between Disco and MGs as well as the local electricity market. The resulting model is a non-linear bi-level optimisation problem which is transformed into a single-level linear problem using Karush–Kuhn–Tucker (KKT) conditions and dual theory. The local electricity market which is modelled in this paper is different from the one studied in [12, 13]. In [12, 13], the distribution network operator receives offers from generators and bids from dispatchable loads in the form of blocks for each hour and clears the local market based on social welfare maximisation. In fact, the power exchange between decision makers is done based on their bids and offers. However, in the proposed local electricity market in this paper, the power exchange between Disco and MGs is done based on local electricity price which couples these decision makers. Details of the proposed local electricity market are described in Section 2.

Main contributions of this paper are as follows:

(i) Proposing a bi-level optimisation approach that models the hierarchical decision-making framework of ADG in which Disco and MGs optimise their respective objective functions independently and in cooperation with each other.
(ii) Modelling a local electricity market in distribution grid level between Disco and MGs.

1.3 Paper organisation

The rest of this paper is organised as follows. Section 2 presents the problem statement. The bi-level problem formulation and its solution methodology are described in Sections 3 and 4, respectively. Numerical results are given in Section 5. Section 6 concludes this paper. Modelling uncertainties and mathematical backgrounds are explained in the Appendix.

2 Problem statement

Fig. 1 shows a general perspective of energy and information interactions among different levels of power system decision makers. In this framework, generation companies (Gencos) are the first-level decision makers. Discos, retailers, and large consumers are the second-level decision makers and MGs are considered as the third-level decision makers. Other participants, due to their functionalities, may be located at each level of this framework. The first- and second-level decision makers cooperate with each

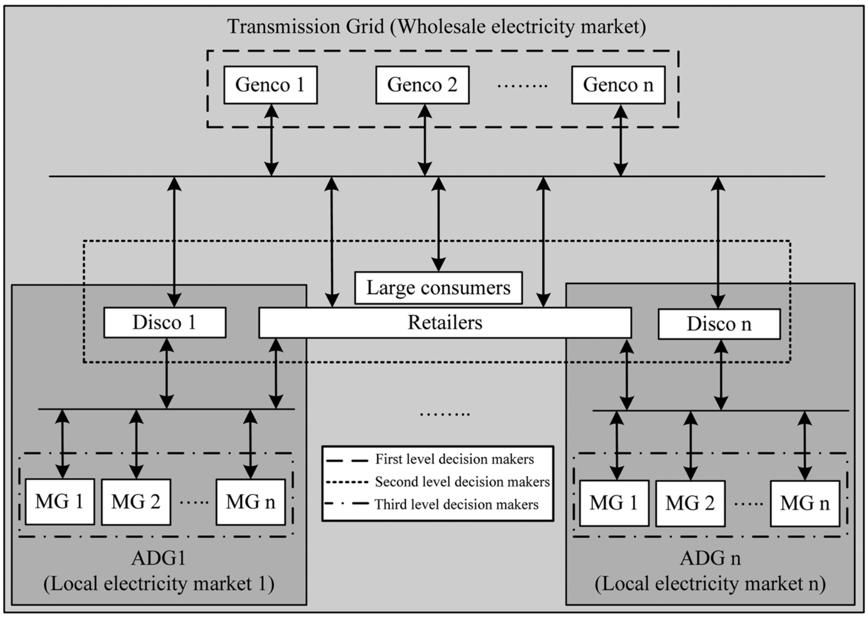

Fig. 1 *General perspective of interactions between different levels of power system decision makers*

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other in a wholesale electricity market, which is managed by the independent system operator. The second- and third-level decision makers cooperate with each other in local electricity markets in ADGs level. In this paper, the cooperation environment in an ADG level between its internal MGs and Disco is investigated (Fig. 2). These decision makers are coupled by local electricity price and exchange power with each other based on this price as illustrated in Fig. 3. For modelling this framework, a bi-level optimisation approach is developed. The bi-level optimisation problem has two levels of decision makers in which the upper level decision maker leads the problem and the lower level decision makers follow the leader strategies. In the proposed model, Disco and MGs are considered as the leader and the followers, respectively. Disco as the upper level decision maker (leader) of the problem is considered as a price taker player in wholesale electricity market receiving electricity at a predetermined price and cooperates with MGs in local electricity market as shown in Fig. 2. Therefore, the cooperation of Disco and MGs has no effect on the wholesale electricity prices. As illustrated in Fig. 3, Disco offers the local electricity prices to the MGs. MGs receive these prices and schedule their resources and decide about the power exchange with Disco. In equilibrium points, the local electricity prices and the amount of power exchanges between Disco and MGs are determined. Moreover, Disco decides about the purchased power from wholesale electricity market and each MG schedules its power generation of DGs, the amount of load curtailment, and charging/discharging power of batteries. On the basis of the amount of demand and characteristics of the resources, MGs can act as a consumer or producer in each hour in local electricity market.

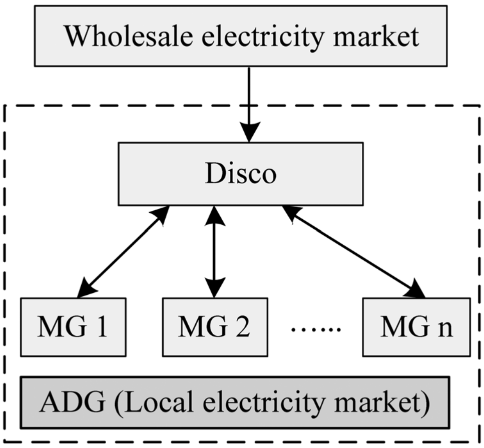

Fig. 2 *Interactions between two levels of decision makers in ADG*

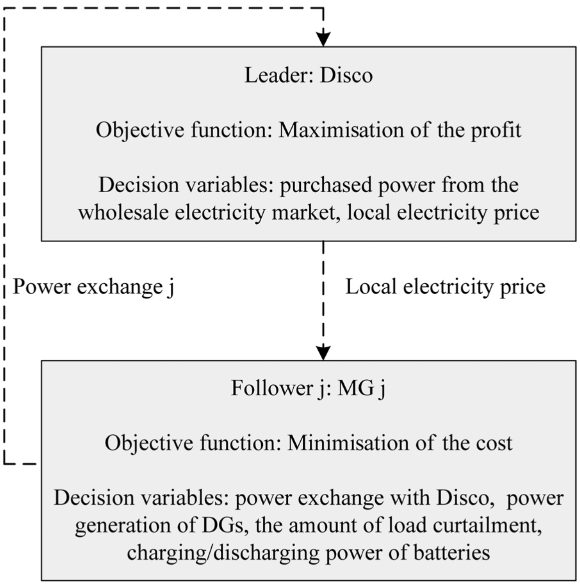

Fig. 3 *Bi-level decision-making framework*

3 Bi-level problem formulation

The bi-level optimisation approach proposed in this paper is formulated as follows

$$\underset{Y}{\text{maximise}} \quad \sum_{t=1}^{T}\sum_{j=1}^{J}\left(\rho_t^{\text{D}}\hat{P}_{j,t}^{\text{D}}\right) - \sum_{t=1}^{T}(\rho_t^{\text{M}}P_t^{\text{M}}) \tag{1}$$

subject to

$$0 \le \rho_t^{\text{D}} \le \rho^{\max}, \quad \forall t \tag{2}$$

$$0 \le P_t^{\text{M}} \le P^{\max}, \quad \forall t \tag{3}$$

$$\sum_{j=1}^{J}\hat{P}_{j,t}^{\text{D}} = P_t^{\text{M}}, \quad \forall t \tag{4}$$

where

$$X \in \arg\left\{ \text{minimise} \quad \sum_{t=1}^{T}\sum_{s=1}^{S_j}\pi_s\left[\rho_t^{\text{D}}P_{j,t,s}^{\text{D}} + C_j^{\text{DG}}P_{j,t,s}^{\text{DG}} + C_{j,t}^{\text{IL}}P_{j,t,s}^{\text{IL}}\right]\right. \tag{5}$$

$$\hat{P}_{j,t}^{\text{D}} = \sum_{s=1}^{S_j}\pi_s P_{j,t,s}^{\text{D}}, \quad \forall t \tag{6}$$

subject to

$$-P_{\max}^{T_j} \le P_{j,t,s}^{\text{D}} \le P_{\max}^{T_j} \quad : \quad \lambda_{j,t,s}^{1},\ \lambda_{j,t,s}^{2}, \quad \forall t,\ s \tag{7}$$

$$P_j^{\text{DG,min}} \le P_{j,t,s}^{\text{DG}} \le P_j^{\text{DG,max}} \quad : \quad \lambda_{j,t,s}^{3},\ \lambda_{j,t,s}^{4}, \quad \forall t,\ s \tag{8}$$

$$P_{j,t,s}^{\text{DG}} - P_{j,t-1,s}^{\text{DG}} \le \text{RUP}_j^{\text{DG}} \quad : \quad \lambda_{j,t,s}^{5}, \quad \forall t > 1,\ s \tag{9}$$

$$P_{j,t,s}^{\text{DG}} - P_j^{\text{DG,ini}} \le \text{RUP}_j^{\text{DG}} \quad : \quad \lambda_{j,t,s}^{6}, \quad \forall t = 1,\ s \tag{10}$$

$$P_{j,t-1,s}^{\text{DG}} - P_{j,t,s}^{\text{DG}} \le \text{RDN}_j^{\text{DG}} \quad : \quad \lambda_{j,t,s}^{7}, \quad \forall t > 1,\ s \tag{11}$$

$$P_j^{\text{DG,ini}} - P_{j,t,s}^{\text{DG}} \le \text{RDN}_j^{\text{DG}} \quad : \quad \lambda_{j,t,s}^{8}, \quad \forall t = 1,\ s \tag{12}$$

$$0 \le P_{j,t,s}^{\text{IL}} \le P_{j,t}^{\text{IL,max}} \quad : \quad \lambda_{j,t,s}^{9},\ \lambda_{j,t,s}^{10}, \quad \forall t,\ s \tag{13}$$

$$0 \le P_{j,t,s}^{\text{ch}} \le P_j^{\text{batt,max}} \quad : \quad \lambda_{j,t,s}^{11},\ \lambda_{j,t,s}^{12}, \quad \forall t,\ s \tag{14}$$

$$0 \le P_{j,t,s}^{\text{dch}} \le P_j^{\text{batt,max}} \quad : \quad \lambda_{j,t,s}^{13},\ \lambda_{j,t,s}^{14}, \quad \forall t,\ s \tag{15}$$

$$E_j^{\min} \le E_{j,t,s} \le E_j^{\max} \quad : \quad \lambda_{j,t,s}^{15},\ \lambda_{j,t,s}^{16}, \quad \forall t,\ s \tag{16}$$

$$E_{j,t,s} = E_{j,t-1,s} + \eta_j^{\text{ch}}P_{j,t,s}^{\text{ch}} - \frac{P_{j,t,s}^{\text{dch}}}{\eta_j^{\text{dch}}} \quad : \quad \lambda_{j,t,s}^{17}, \quad \forall t > 1,\ s \tag{17}$$

$$E_{j,t,s} = E_j^{\text{ini}} + \eta_j^{\text{ch}}P_{j,t,s}^{\text{ch}} - \frac{P_{j,t,s}^{\text{dch}}}{\eta_j^{\text{dch}}} \quad : \quad \lambda_{j,t,s}^{18}, \quad \forall t = 1,\ s \tag{18}$$

(see (19) at bottom of next page)

In (1)–(19), $\boldsymbol{Y} = \left[\rho_t^{\text{D}} \quad P_t^{M}\right]$ and $\boldsymbol{X} = \left[P_{j,t,s}^{\text{D}} \quad P_{j,t,s}^{\text{DG}} \quad P_{j,t,s}^{\text{IL}} \quad P_{j,t,s}^{\text{ch}} \quad P_{j,t,s}^{\text{dch}} E_{j,t,s}\right]$ are Disco and MGs decision variable vectors, respectively.

$P^{DG}_{j,t,s}$ and $E_{j,t,s}$ are decision variables with time-dependence in the follower problems. Equations (1)–(4) describe the Disco problem and (5)–(19) describe the MGs problems. The term $\rho^D_t \hat{P}^D_{j,t}$ which plays an important role in both objective functions of the upper and lower levels, couples the two levels and determines the financial trade between the levels' decision makers. The objective of the Disco is to maximise its profit, which is modelled by (1). The first term of this equation is the revenue from the power exchange with MGs. $\hat{P}^D_{j,t} > 0$ shows that Disco is selling power to MG j and $\hat{P}^D_{j,t} < 0$ shows that Disco is purchasing power from MG j also, $\hat{P}^D_{j,t} = 0$ shows that there is not any power exchange between Disco and MG j. The second term shows the cost of purchased power from the wholesale electricity market by Disco. Equation (2) is used to limit the price of power exchange between Disco and MGs. The purchased power from the wholesale market is limited by (3). Since considering the distribution network topology increases the complexity of investigating the decision makers' behaviour in the proposed model, it is not considered and all MGs are assumed to be connected to a common bus [5, 7, 9]. In fact, the power losses of distribution network is neglected and (4) is used to model the power balance constraint of Disco in which the sum of power exchange with MGs is equal to the purchased power from the wholesale electricity market.

The objective function of each MG is modelled by (5) including the cost of power exchange with Disco, the cost of DG power generation, and the cost of load curtailment contract. In this paper, the customer of each MG can sign contract with it to be curtailed. For this purpose, a bid-based mechanism for load curtailment is considered wherein the customers submit their offers in terms of the maximum amount of load curtailment and associated price on an hourly basis as proposed in [14, 15]. Equation (6) describes the expected value of the power exchange between Disco and each MG which is used in the Disco problem. Equation (7) is used to limit the power exchange between Disco and MG. The power generation of DG is limited by (8). Ramp down and ramp up limits of DG are modelled by (9)–(12). The amount of load curtailment is limited by (13). Charging and discharging power and also energy storage of the battery are limited by (14)–(16). The value of energy storage in battery at each time step is dependent to its value in the previous time step and the value of charging/discharging power in present time step which is modelled by (17) and (18) [16]. Although (17) and (18) do not prevent charging and discharging of the battery at the same time, this point will be considered in the optimal solution of the bi-level model [16]. In the proposed model, each MG is modelled as a single bus without considering the topology, wires, and transformers [5, 17]. Therefore, the power balance constraint of each MG is described by (19) in which the sum of power generation of wind turbine and DG, the amount of load curtailment, the power exchange with Disco, and discharging power of battery are equal to the demand and charging power of battery. It is assumed that each MG forecasts the output power of its wind turbine. Moreover, the uncertainties of power generation of wind turbine as well as the demand of each MG are modelled as described in Appendix 1. $\lambda^1_{j,t,s} - \lambda^{19}_{j,t,s}$ are the dual variables defined for the follower problems.

Since in (1) two variables ρ^D_t and $\hat{P}^D_{j,t}$ are multiplied, the optimisation problems (1)–(19) are non-linear bi-level one. The solution methodology for solving this optimisation problem is presented in the next section.

4 Solution methodology

To solve the non-linear bi-level optimisation problems, different approaches are proposed [18]. In the literature, KKT conditions and dual theory are proposed as the most common and exact solutions for these problems [9, 19–21]. However, due to presence of decision variables with time-dependence in the follower problem of the proposed model, an unique approach for solving the bi-level model based on KKT condition and dual theory is used, where the non-linear bi-level problem is transformed to a single-level mixed-integer linear problem (MILP). The problem transformation is done in two steps as follows:

- In bi-level optimisation problem, the variables of the leader problem are considered as parameters in the follower problems. Therefore, ρ^D_t is considered as a parameter in the follower problems. Since the follower problems are linear and continuous and thus are convex, the follower problems are replaced with their KKT conditions as described in Appendix 2.
- The resulting model from the previous step is a single-level mixed-integer non-linear problem. To linearise the model, the non-linear expression in the objective function is replaced with linear ones using dual theory. Details of this step are described in Appendix 3.

The resulting single-level MILP is described as follows (see (20))

subject to

$$0 \le \rho^D_t \le \rho^{max}, \quad \forall t \tag{21}$$

$$0 \le P^M_t \le P^{max}, \quad \forall t \tag{22}$$

$$\sum_{j=1}^{J} \hat{P}^D_{j,t} = P^M_t, \quad \forall t \tag{23}$$

$$\rho^D_t - \lambda^1_{j,t,s} + \lambda^2_{j,t,s} - \lambda^{19}_{j,t,s} = 0, \quad \forall j,\ t,\ s \tag{24}$$

(see (25) at the bottom of next page)

$$C^{IL}_{j,t} - \lambda^9_{j,t,s} + \lambda^{10}_{j,t,s} - \lambda^{19}_{j,t,s} = 0, \quad \forall j,\ t,\ s \tag{26}$$

$$-\lambda^{11}_{j,t,s} + \lambda^{12}_{j,t,s} + \eta^{ch}_j \lambda^{17}_{j,t,s}\Big|_{t>1} + \eta^{ch}_j \lambda^{18}_{j,t,s}\Big|_{t=1} + \lambda^{19}_{j,t,s} = 0, \quad \forall j,\ t,\ s \tag{27}$$

$$-\lambda^{13}_{j,t,s} + \lambda^{14}_{j,t,s} - \frac{\lambda^{17}_{j,t,s}}{\eta^{dch}_j}\Bigg|_{t>1} - \frac{\lambda^{18}_{j,t,s}}{\eta^{dch}_j}\Bigg|_{t=1} - \lambda^{19}_{j,t,s} = 0, \quad \forall j,\ t,\ s \tag{28}$$

$$\left. P^{WT}_{j,t,s} + P^{DG}_{j,t,s} + P^{IL}_{j,t,s} + P^{D}_{j,t,s} + P^{dch}_{j,t,s} = P^{demand}_{j,t,s} + P^{ch}_{j,t,s} : \quad \lambda^{19}_{j,t,s}, \quad \forall t,\ s \right\} \quad \forall j \tag{19}$$

$$\max_{Y} \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{s=1}^{S_j} \pi_s \begin{bmatrix} -C^{DG}_j P^{DG}_{j,t,s} - C^{IL}_{j,t} P^{IL}_{j,t,s} - P^{T_j}_{max}(\lambda^1_{j,t,s} + \lambda^2_{j,t,s}) + P^{DG,min}_j \lambda^3_{j,t,s} \\ -P^{DG,max}_j \lambda^4_{j,t,s} - RUP^{DG}_j \lambda^5_{j,t,s} - (RUP^{DG}_j + P^{DG,ini}_j)\lambda^6_{j,t,s} \\ -RDN^{DG}_j \lambda^7_{j,t,s} - (RDN^{DG}_j - P^{DG,ini}_j)\lambda^8_{j,t,s} - P^{IL,max}_{j,t}\lambda^{10}_{j,t,s} \\ -P^{batt,max}_j(\lambda^{12}_{j,t,s} + \lambda^{14}_{j,t,s}) + E^{min}_j \lambda^{15}_{j,t,s} - E^{max}_j \lambda^{16}_{j,t,s} + E^{ini}_j \lambda^{18}_{j,t,s} \\ +(-P^{WT}_{j,t,s} + P^{demand}_{j,t,s})\lambda^{19}_{j,t,s} \end{bmatrix} - \sum_{t=1}^{T} \rho^M_t P^M_t \tag{20}$$

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Table 1 Characteristics of MGs' resources

Resource		MG 1	MG 2	MG 3
DG [15]	C_j^{DG}, \$/MWh	37	40	35
	$P_j^{\text{DG, ini}}$, MW	0	0	0
	$P_j^{\text{DG, min}}$, MW	0	0	0
	$P_j^{\text{DG, max}}$, MW	4	5	5.5
	RDN_j^{DG}, MW/h	1	1.25	1.375
	RUP_j^{DG}, MW/h	1	1.25	1.375
battery [22]	E_j^{ini}, MWh	1	0	0
	E_j^{min}, MWh	1	0	0
	E_j^{max}, MWh	2.5	0	0
	$P_j^{\text{batt, max}}$, MW	0.5	0	0
	η_j^{ch}	0.95	0	0
	η_j^{dch}	0.95	0	0
wind turbine [23]	Capacity, MW	1.1	0	0

$$\begin{cases} -\lambda_{j,t,s}^{15} + \lambda_{j,t,s}^{16} - \lambda_{j,t,s}^{17} + \lambda_{j,t+1,s}^{17} = 0, & \forall j,\ 1 < t < T,\ s \\ -\lambda_{j,t,s}^{15} + \lambda_{j,t,s}^{16} + \lambda_{j,t+1,s}^{17} - \lambda_{j,t,s}^{18} = 0, & \forall j,\ t = 1,\ s \\ -\lambda_{j,t,s}^{15} + \lambda_{j,t,s}^{16} - \lambda_{j,t,s}^{17} = 0, & \forall j,\ t = T,\ s \end{cases} \tag{29}$$

$$E_{j,t,s} = E_{j,t-1,s} + \eta_j^{\text{ch}} P_{j,t,s}^{\text{ch}} - \frac{P_{j,t,s}^{\text{dch}}}{\eta_j^{\text{dch}}} \ : \ \lambda_{j,t,s}^{17}, \quad \forall j,\ t > 1,\ s \tag{30}$$

$$E_{j,t,s} = E_j^{\text{ini}} + \eta_j^{\text{ch}} P_{j,t,s}^{\text{ch}} - \frac{P_{j,t,s}^{\text{dch}}}{\eta_j^{\text{dch}}} \ : \ \lambda_{j,t,s}^{18}, \quad \forall j,\ t = 1,\ s \tag{31}$$

$$P_{j,t,s}^{\text{WT}} + P_{j,t,s}^{\text{DG}} + P_{j,t,s}^{\text{IL}} + P_{j,t,s}^{\text{D}} + P_{j,t,s}^{\text{dch}} = P_{j,t,s}^{\text{demand}} + P_{j,t,s}^{\text{ch}} : \ \lambda_{j,t,s}^{19}, \quad \forall j,\ t,\ s \tag{32}$$

$$C_{j,t,s}^{i} \ge 0 \quad \forall i = 1, 2, \ldots, 16, \quad \forall j,\ t,\ s \tag{33}$$

$$\lambda_{j,t,s}^{i} \ge 0 \quad \forall i = 1, 2, \ldots, 16, \quad \forall j,\ t,\ s \tag{34}$$

$$\lambda_{j,t,s}^{17},\ \lambda_{j,t,s}^{18},\ \lambda_{j,t,s}^{19}, \quad \forall j,\ t,\ s \quad \text{unrestricted} \tag{35}$$

$$C_{j,t,s}^{i} \le M_1 U_{j,t,s}^{i}, \quad \forall i = 1, 2, \ldots, 16, \quad \forall j,\ t,\ s \tag{36}$$

$$\lambda_{j,t,s}^{i} \le M_2 (1 - U_{j,t,s}^{i}) \quad \forall i = 1, 2, \ldots, 16, \quad \forall j,\ t,\ s \tag{37}$$

The first term of (1), which is non-linear, is replaced with linear expressions. Therefore, (1) is rewritten as (20). Equations (21)–(23) are the same equations as (2)–(4) and other ones are described in Appendix 2.

5 Numerical results

A hypothetical distribution grid including three MGs is considered as the case study. The characteristics of MGs' resources are described in Table 1 [15, 22, 23]. Modified power demand of three types of consumers is considered as the MGs' demand as given in Table 2 [24]. In addition, the wholesale electricity prices which are collected from Nord Pool Spot market on Monday 2 February 2015 [25] and the load curtailment contract price [15] are given in Table 2. It is assumed that the load curtailment contract price is equal for all customers in MGs. The maximum amount of load curtailment is 10% of demand for each MG in each time step [15]. The maximum limit for purchasing power from the wholesale electricity market and maximum limit for power exchange between Disco and MGs are 50 and 10 MW, respectively [10]. The maximum limit for price of power exchange between Disco and MGs is assumed to be 90 \$/MWh. The forecasted wind speed is obtained from [23]. Since MG 1 have wind turbine, the uncertainty of wind speed and demand is considered for it. Moreover, the uncertainty of demand is only considered for MG 2 and MG 3.

The results are presented in three sections. The operational manner of Disco and MGs in the cooperation environment in local electricity market is investigated in the early one. Afterwards, operation of the ADG is investigated by centralised model, which relies on Disco decisions to schedule MGs' resources and purchase power from market. Finally, the results of two models are compared with each other.

Table 2 Time-dependent input parameters

Time	Demand, MW			Wholesale electricity price, \$/MWh	Load curtailment contract price, \$/MWh
	MG 1	MG 2	MG 3		
1	1.63	2.09	2.69	37.375	50
2	1.56	2.01	2.52	36.7	30
3	1.66	1.9	2.39	36.465	35.5
4	1.69	1.7	2.22	36.5	41
5	1.76	1.59	2.54	36.67	50
6	1.88	1.77	3.21	38.25	65
7	2.48	2.37	4.21	41.43	68
8	3.55	3.09	5.64	59.735	75
9	4.54	3.24	6.52	66.47	78
10	5.54	3.42	7.19	66.53	76
11	5.88	3.78	7.19	64.88	65
12	6.18	3.78	6.52	49.28	85
13	6.38	4.15	7.19	45.03	87
14	6.34	4.48	7.19	46.345	80
15	6.05	4.56	6.87	45.5	70
16	5.88	4.13	6.54	44.616	65
17	6.2	3.8	5.52	63.82	65
18	6.87	4.89	4.66	76.22	80
19	5.52	6.28	4.21	77.8	80
20	4.88	7.35	3.87	53	85
21	4.18	6.64	3.52	43	89
22	3.54	5.59	3.2	40.6	75
23	2.54	4.5	3	39.4	65
24	1.84	3.7	2.86	38.3	65

Table 3 Local electricity prices determined in cooperation environment

Time	ρ_t^{D}, \$/MWh	Time	ρ_t^{D}, \$/MWh
1	90	13	87
2	40	14	80
3	37	15	70
4	37	16	65
5	37	17	65
6	35	18	80
7	35	19	80
8	41	20	85
9	49	21	49
10	76	22	37
11	80	23	37
12	85	24	37

5.1 *Operation results in bi-level model*

In this section, the results of cooperation between Disco and MGs are described. As mentioned before, the power exchange between these decision makers is done based on local electricity price which couples them. The results confirmed that this price is

$$\begin{cases} C_j^{\text{DG}} - \lambda_{j,t,s}^{3} + \lambda_{j,t,s}^{4} + \lambda_{j,t,s}^{5} - \lambda_{j,t+1,s}^{5} + \lambda_{j,t+1,s}^{7} - \lambda_{j,t,s}^{7} - \lambda_{j,t,s}^{19} = 0, & \forall j,\ 1 < t < T,\ s \\ C_j^{\text{DG}} - \lambda_{j,t,s}^{3} + \lambda_{j,t,s}^{4} - \lambda_{j,t+1,s}^{5} + \lambda_{j,t,s}^{6} + \lambda_{j,t+1,s}^{7} - \lambda_{j,t,s}^{8} - \lambda_{j,t,s}^{19} = 0, & \forall j,\ t = 1,\ s \\ C_j^{\text{DG}} - \lambda_{j,t,s}^{3} + \lambda_{j,t,s}^{4} + \lambda_{j,t,s}^{5} - \lambda_{j,t,s}^{7} - \lambda_{j,t,s}^{19} = 0 & \forall j,\ t = T,\ s \end{cases} \tag{25}$$

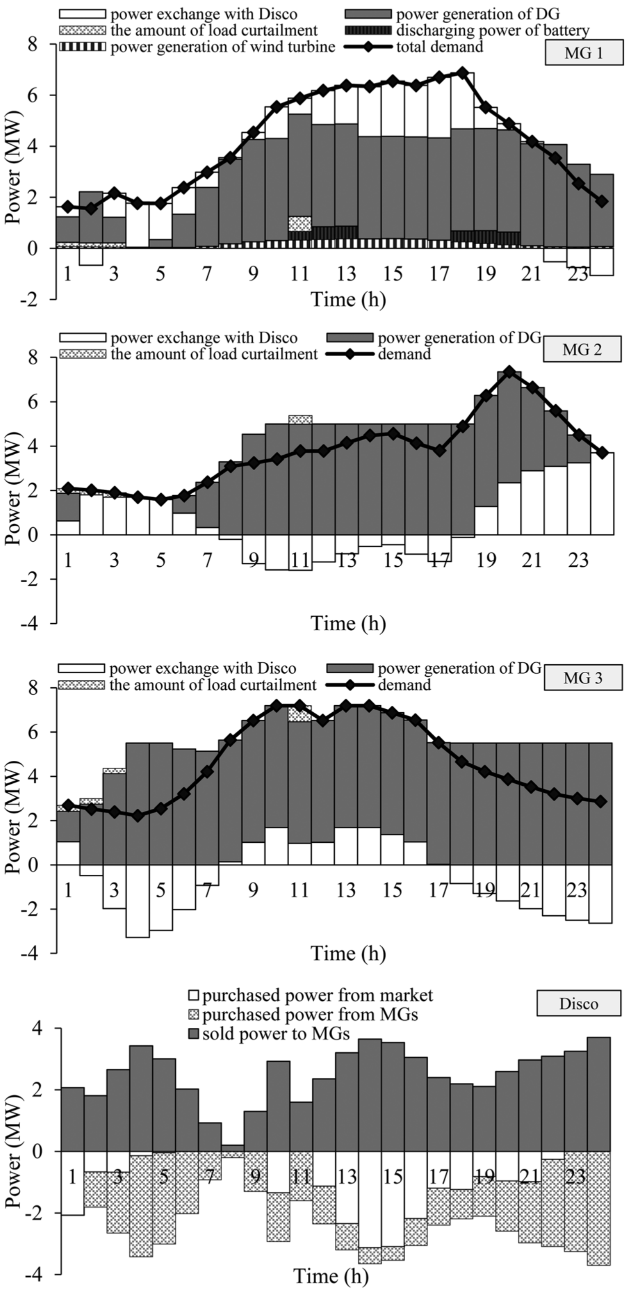

Fig. 4 *Operation results for Disco and MGs in bi-level model*

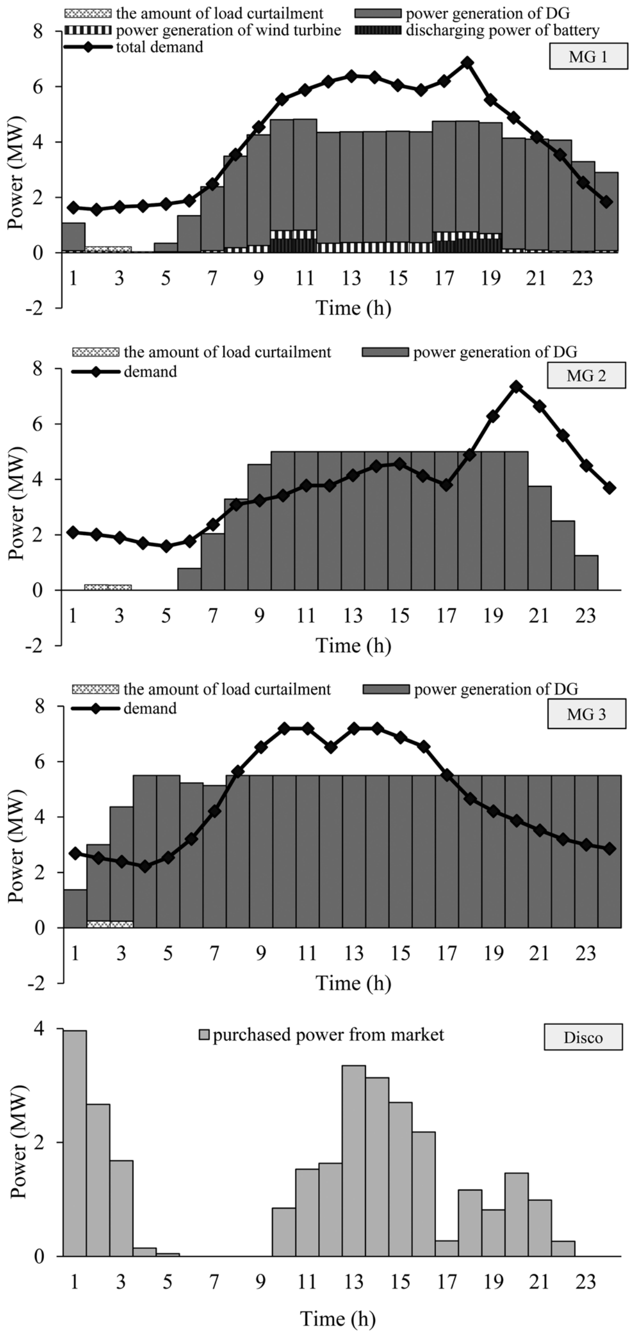

Fig. 6 *Operation results in centralised model*

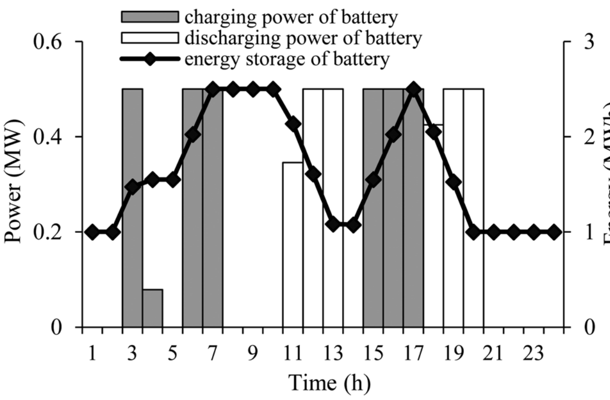

Fig. 5 *Dynamic behaviour of battery in bi-level model*

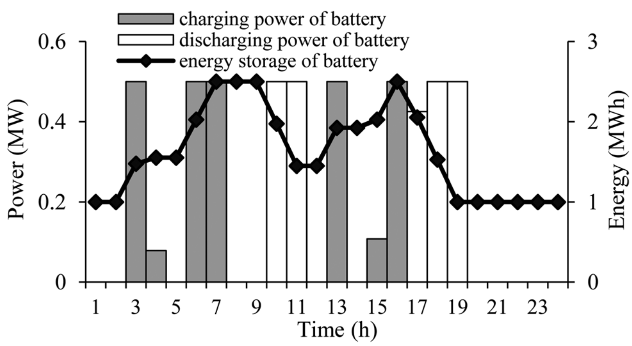

Fig. 7 *Dynamic behaviour of battery in centralised model*

Table 4 Comparison operation results in bi-level and centralised models

	Centralised model				Bi-level model			
	MG1	MG2	MG3	Disco	MG1	MG2	MG3	Disco
total power generation of DG, MW	70.29	73.16	123.11	–	73.29	74.41	123.11	–
total load curtailment, MW	0.322	0.391	0.491	–	1.073	0.978	1.479	–
total charging power of battery, MW	2.68	–	–	–	3.079	–	–	–
total discharging power of battery, MW	2.425	–	–	–	2.77	–	–	–
total purchased power from market, MW	–	–	–	28.86	–	–	–	22.32

dependent on the generation cost of MGs' DGs, load curtailment contract price, and characteristics of MGs' resources including ramp rate limits etc. Moreover, the dynamic behaviour of these decision makers in the time period of problem has important effects on the results. The local electricity prices between Disco and MGs are shown in Table 3. In hour 1, the local electricity price is equal to the price cap and in hours 2–7 and 22–24, the local electricity prices are equal to the generation cost of MGs' DGs. Moreover, in hours 10 and 12–20, the local electricity prices are equal to the load curtailment contract prices and in other hours are determined with respect to dynamic behaviour of MGs resources. The operation results of Disco and MGs are illustrated in Fig. 4. In MG1, the sum of demand and charging power of battery is considered as the total demand. Each MG schedules its resources and cooperates with Disco with respect to the local prices. In hours 1–3 and 11, MGs curtail their demand because the local prices are greater than the load curtailment contract prices.

MG 1 in hours 2, and 22–24, MG 2 in hours 8–18, MG3 in hours 2–7, and 18–24 sell power to the Disco (other MGs) because the demand of MGs is low and the local prices are equal to or greater than the generation cost of MGs' DGs in respective hours.

Owing to low generation of MGs' DGs arises from ramp rate limits in hour 1, Disco increases the local price to the maximum value which, in turn, increases the profit. In this hour, Disco purchases the required demand of the system from the wholesale market.

Since in hours 6–9, 23, and 24, the local prices are lower than the wholesale prices and in hour 11, the load curtailment contract price is lower than local price, Disco only manages power exchange between MGs without purchasing power from the market.

In other hours, due to high demand of MGs, Disco purchases power from the wholesale market in addition to managing power exchange between MGs. In such case, Disco increases the local prices to a value higher than the wholesale prices.

The dynamic behaviour of MG1' battery is illustrated in Fig. 5. Owing to low local prices in hours 3–7 (except 5), and 15–17, the battery is charged while in hours 11–13, and 18–20 when the local prices are high it is discharged. This operational manner of battery reduces the MG 1 operation cost.

5.2 Operation results in centralised model

In centralised model, Disco is responsible for optimal scheduling of MGs' resources and purchases power from the market with respect to the wholesale prices. The operation results of this model are illustrated in Fig. 6. In hours 2 and 3, Disco curtails demand because the load curtailment contract prices are lower than the wholesale prices. In hours 6–9, 23, and 24, Disco purchases no power from the market and meet the demand with optimal scheduling of DGs. In hours 10–22, due to high demand, and in hours 1–5, due to low wholesale prices, Disco purchases required power from the wholesale market.

The dynamic behaviour of battery in this model is illustrated in Fig. 7. It reveals that battery is charged in hours 3–7 (except 5), and 13–16 (except 14) due to low wholesale prices and is discharged in hours 10, 11, and 17–19 due to high wholesale prices.

5.3 Comparison between bi-level and centralised models

In the previous sections, the operation results of two models were presented. Comparison of the results of them lead to the following conclusions:

- In the proposed model, utilisation of resources is increased in comparison with the centralised model as illustrated in Table 4. Moreover, the purchased power from the market is decreased in bi-level model. These are occurring due to cooperation environment between decision makers in bi-level model.
- The total operation cost in bi-level and centralised models are 12,924.8$ and 11,243$, respectively. Therefore, the operation cost of system in centralised model is lower than the bi-level model. This difference could be justified by means of the local prices which are higher than the wholesale prices for some hours.

6 Conclusion

Modelling the cooperation between Disco and MGs was addressed in this paper. This framework is modelled as a bi-level optimisation problem in which the objective function of the upper level problem is maximising the profit of Disco, and the objective function of the lower level problems is minimising the cost of MGs. The price and amount of power exchange between Disco and MGs are considered as the interactions' decision variables. To solve the proposed model, KKT conditions and dual theory were used. Optimal decision making of Disco and MGs shows the effectiveness of the model and its solution methodology. Moreover, the remarkable conclusions from the results are as follows:

- The combination of DERs in each MG determines its decision making to cooperate with others.
- The dynamic behaviour of MGs' resources has the important effects on the local prices.

Inclusion of distribution network topology in the problem modelling and investigating its impact on the decision makers' behaviours are considered as the future work.

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8 Appendix

8.1 Appendix 1: Modelling uncertainties

Wind speed uncertainty: To model the uncertainty of wind speed, a five-interval Rayleigh probability density function (PDF) is used as illustrated in Fig. 8 [23, 26]. The forecasted wind speed in each time step is considered as the mean value of Rayleigh PDF (v_{mean}). Moreover, the probability of each interval (scenario) is calculated through integration of the mentioned function ((38) and (39)).

$$\pi_s^{\text{WT}} = \int_{\underline{v}_s}^{\bar{v}_s} f(v)\,\mathrm{d}v \tag{38}$$

$$f(v) = \frac{2v}{c^2}\mathrm{e}^{-(v/c)^2} \tag{39}$$

where $f(v)$ is Rayleigh PDF and π_s^{WT}, $\underline{v}_s$, and $\overline{v_s}$ are probability of each scenario and lower and upper limits of wind speed in each scenario, respectively. In addition, v and c are wind speed and scale index. Then, the forecasted power generation of wind turbine

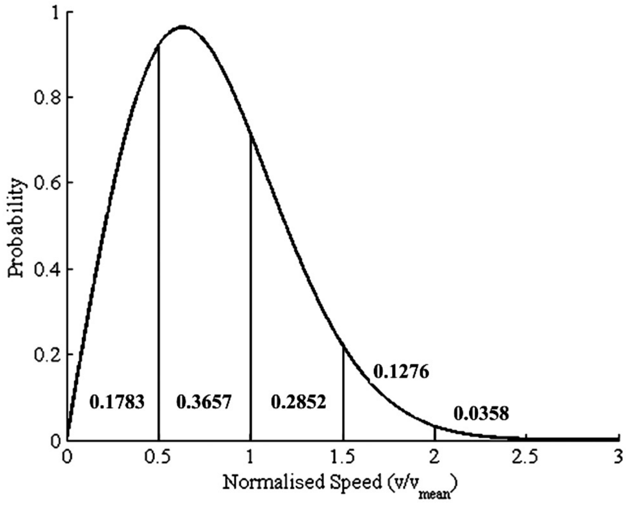

Fig. 8 *Rayleigh PDF for wind speed*

in each scenario (P_s^{WT}) is calculated using (40)

$$P_s^{\text{WT}} = \begin{cases} 0, & 0 \le v_{as} < v_{\text{i}} \text{ and } v_{as} > v_{\text{o}} \\ P_{\text{r}}((v_{as} - v_{\text{i}})/(v_{\text{r}} - v_{\text{i}})), & v_{\text{i}} \le v_{as} \le v_{\text{r}} \\ P_{\text{r}}, & v_{\text{r}} \le v_{as} \le v_{\text{o}} \end{cases} \tag{40}$$

where v_{as} is the average value of wind speed in each scenario. For instance, in scenario one the normalised wind speed between 0 and 0.5 gives rise to $v_{a1} = 0.25v_{\text{mean}}$. Moreover, P_{r}, v_{i}, v_{o}, and v_{r} are rated power, cut-in wind speed, cut-out wind speed, and rated wind speed of wind turbine, respectively.

Demand uncertainty: A seven-interval normal PDF is employed to model the demand uncertainty as illustrated in Fig. 9 [23, 26]. The probability of each interval (scenario) is calculated through (41)

$$\pi_s^{\text{demand}} = \frac{1}{\sigma_d\sqrt{2\pi}}\int_{\underline{d}_s}^{\bar{d}_s} \mathrm{e}^{-(x-\mu_d)^2/2\sigma_d^2}\,\mathrm{d}x \tag{41}$$

where π_s^{demand}, $\underline{d}_s$, and $\overline{d_s}$ are probability of each scenario and lower and upper limits of demand in each scenario, respectively. In addition, μ_d is the mean value of the normal PDF which is equal to the forecasted demand in each time step and σ_d is the standard deviation of normal PDF which is equal to the 5% of forecasted demand in each time step.

Therefore, for uncertainties modelling of wind turbine power generation and demand, five and seven scenarios are considered in each time step, respectively. Since some MGs have wind turbine, there are two different sets of scenarios for MGs. MGs without wind turbine have the set of scenarios (S_j) only including demand

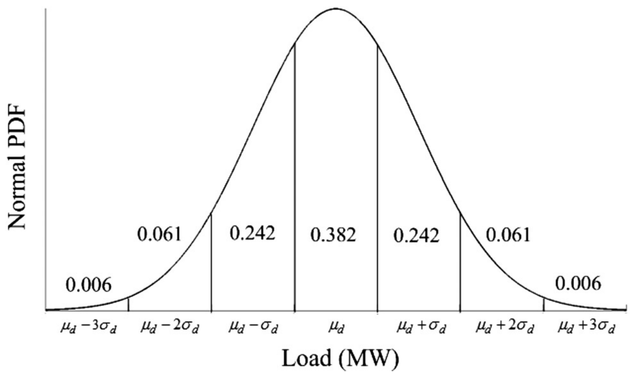

Fig. 9 *Normal PDF for demand*

uncertainty (seven scenarios) and $\pi_s = \pi_s^{\text{demand}}$. Moreover, MGs with wind turbine have the set of scenarios (S_j) including demand and wind speed uncertainties (35 scenarios) and $\pi_s = \pi_s^{\text{demand}} \times \pi_s^{\text{WT}}$. For these MGs, the set of scenarios is obtained from two-stage scenario tree model [23].

8.2 Appendix 2: KKT conditions

Transformation of the non-linear bi-level optimisation problem to the non-linear single-level one is done using KKT conditions [19–21]. At first, all non-equality constraints of the follower problems are rewritten as greater than or equal to zero constraints as follows

$$C^1_{j,t,s} = P^{\text{D}}_{j,t,s} + P^{T_j}_{\max} \geq 0 \quad : \quad \lambda^1_{j,t,s}, \quad \forall j,\ t,\ s \tag{42}$$

$$C^2_{j,t,s} = P^{T_j}_{\max} - P^{\text{D}}_{j,t,s} \geq 0 \quad : \quad \lambda^2_{j,t,s}, \quad \forall j,\ t,\ s \tag{43}$$

$$C^3_{j,t,s} = P^{\text{DG}}_{j,t,s} - P^{\text{DG,min}}_{j} \geq 0 \quad : \quad \lambda^3_{j,t,s}, \quad \forall j,\ t,\ s \tag{44}$$

$$C^4_{j,t,s} = P^{\text{DG,max}}_{j} - P^{\text{DG}}_{j,t,s} \geq 0 \quad : \quad \lambda^4_{j,t,s}, \quad \forall j,\ t,\ s \tag{45}$$

$$C^5_{j,t,s} = \text{RUP}^{\text{DG}}_j - P^{\text{DG}}_{j,t,s} + P^{\text{DG}}_{j,t-1,s} \geq 0 \quad : \quad \lambda^5_{j,t,s}, \quad \forall j,\ t > 1,\ s \tag{46}$$

$$C^6_{j,t,s} = \text{RUP}^{\text{DG}}_j - P^{\text{DG}}_{j,t,s} + P^{\text{DG,ini}}_{j} \geq 0 \quad : \quad \lambda^6_{j,t,s}, \quad \forall j,\ t = 1,\ s \tag{47}$$

$$C^7_{j,t,s} = \text{RDN}^{\text{DG}}_j - P^{\text{DG}}_{j,t-1,s} + P^{\text{DG}}_{j,t,s} \geq 0 \quad : \quad \lambda^7_{j,t,s}, \quad \forall j,\ t > 1,\ s \tag{48}$$

$$C^8_{j,t,s} = \text{RDN}^{\text{DG}}_j - P^{\text{DG,ini}}_{j} + P^{\text{DG}}_{j,t,s} \geq 0 \quad : \quad \lambda^8_{j,t,s}, \quad \forall j,\ t = 1,\ s \tag{49}$$

$$C^9_{j,t,s} = P^{\text{IL}}_{j,t,s} \geq 0 \quad : \quad \lambda^9_{j,t,s}, \quad \forall j,\ t,\ s \tag{50}$$

$$C^{10}_{j,t,s} = P^{\text{IL,max}}_{j,t} - P^{\text{IL}}_{j,t,s} \geq 0 \quad : \quad \lambda^{10}_{j,t,s}, \quad \forall j,\ t,\ s \tag{51}$$

$$C^{11}_{j,t,s} = P^{\text{ch}}_{j,t,s} \geq 0 \quad : \quad \lambda^{11}_{j,t,s}, \quad \forall j,\ t,\ s \tag{52}$$

$$C^{12}_{j,t,s} = P^{\text{batt,max}}_{j} - P^{\text{ch}}_{j,t,s} \geq 0 \quad : \quad \lambda^{12}_{j,t,s}, \quad \forall j,\ t,\ s \tag{53}$$

$$C^{13}_{j,t,s} = P^{\text{dch}}_{j,t,s} \geq 0 \quad : \quad \lambda^{13}_{j,t,s}, \quad \forall j,\ t,\ s \tag{54}$$

$$C^{14}_{j,t,s} = P^{\text{batt,max}}_{j} - P^{\text{dch}}_{j,t,s} \geq 0 \quad : \quad \lambda^{14}_{j,t,s}, \quad \forall j,\ t,\ s \tag{55}$$

$$C^{15}_{j,t,s} = E_{j,t,s} - E^{\min}_{j} \geq 0 \quad : \quad \lambda^{15}_{j,t,s}, \quad \forall j,\ t,\ s \tag{56}$$

$$C^{16}_{j,t,s} = E^{\max}_{j} - E_{j,t,s} \geq 0 \quad : \quad \lambda^{16}_{j,t,s} \quad \forall j,\ t,\ s \tag{57}$$

$$E_{j,t,s} - E_{j,t-1,s} - \eta_j^{\text{ch}} P^{\text{ch}}_{j,t,s} + \frac{P^{\text{dch}}_{j,t,s}}{\eta_j^{\text{dch}}} = 0 \quad : \quad \lambda^{17}_{j,t,s}, \quad \forall j,\ t > 1,\ s \tag{58}$$

$$E_{j,t,s} - E^{\text{ini}}_j - \eta_j^{\text{ch}} P^{\text{ch}}_{j,t,s} + \frac{P^{\text{dch}}_{j,t,s}}{\eta_j^{\text{dch}}} = 0 \quad : \quad \lambda^{18}_{j,t,s}, \quad \forall j,\ t = 1,\ s \tag{59}$$

$$P^{\text{WT}}_{j,t,s} + P^{\text{DG}}_{j,t,s} + P^{\text{IL}}_{j,t,s} + P^{\text{D}}_{j,t,s} + P^{\text{dch}}_{j,t,s} - P^{\text{demand}}_{j,t,s} - P^{\text{ch}}_{j,t,s} = 0 \quad : \lambda^{19}_{j,t,s}, \quad \forall j,\ t,\ s \tag{60}$$

For each follower problem (5)–(19), the Lagrangian function is as follows (see (61))

KKT conditions including four sets of equations are illustrated as follows.

Stationarity

$$\frac{\partial L^j}{\partial P^{\text{D}}_{j,t,s}} = \rho_t^{\text{D}} - \lambda^1_{j,t,s} + \lambda^2_{j,t,s} - \lambda^{19}_{j,t,s} = 0 \tag{62}$$

$$\frac{\partial L^j}{\partial P^{\text{DG}}_{j,t,s}} = C_j^{\text{DG}} - \lambda^3_{j,t,s} + \lambda^4_{j,t,s} + \lambda^5_{j,t,s}\Big|_{t>1} - \lambda^5_{j,t+1,s} + \lambda^6_{j,t,s}\Big|_{t=1} + \lambda^7_{j,t+1,s} - \lambda^7_{j,t,s}\Big|_{t>1} - \lambda^8_{j,t,s}\Big|_{t=1} - \lambda^{19}_{j,t,s} = 0 \tag{63}$$

$$\frac{\partial L^j}{\partial P^{\text{IL}}_{j,t,s}} = C^{\text{IL}}_{j,t} - \lambda^9_{j,t,s} + \lambda^{10}_{j,t,s} - \lambda^{19}_{j,t,s} = 0 \tag{64}$$

$$\frac{\partial L^j}{\partial P^{\text{ch}}_{j,t,s}} = -\lambda^{11}_{j,t,s} + \lambda^{12}_{j,t,s} + \eta_j^{\text{ch}} \lambda^{17}_{j,t,s}\Big|_{t>1} + \eta_j^{\text{ch}} \lambda^{18}_{j,t,s}\Big|_{t=1} + \lambda^{19}_{j,t,s} = 0 \tag{65}$$

$$\frac{\partial L^j}{\partial P^{\text{dch}}_{j,t,s}} = -\lambda^{13}_{j,t,s} + \lambda^{14}_{j,t,s} - \frac{\lambda^{17}_{j,t,s}}{\eta_j^{\text{dch}}}\Bigg|_{t>1} - \frac{\lambda^{18}_{j,t,s}}{\eta_j^{\text{dch}}}\Bigg|_{t=1} - \lambda^{19}_{j,t,s} = 0 \tag{66}$$

$$\frac{\partial L^j}{\partial E_{j,t,s}} = -\lambda^{15}_{j,t,s} + \lambda^{16}_{j,t,s} - \lambda^{17}_{j,t,s}\Big|_{t>1} + \lambda^{17}_{j,t+1,s} - \lambda^{18}_{j,t,s}\Big|_{t=1} = 0 \tag{67}$$

Primal feasibility

Equations (42)–(60) represent the primal feasibility conditions.

Dual feasibility

$$\lambda^i_{j,t,s} \geq 0 \quad \forall i = 1,\ 2,\ \ldots,\ 16, \quad \forall j,\ t,\ s \tag{68}$$

$$\lambda^{17}_{j,t,s},\ \lambda^{18}_{j,t,s},\ \lambda^{19}_{j,t,s} \quad \forall j,\ t,\ s \quad \text{unrestricted} \tag{69}$$

As (42)–(57) are greater than or equal to zero constraints, the respective dual variables are in the same form. Moreover, as (58)–(60) are equality constraints, the respective dual variables are unrestricted in sign.

$$\begin{aligned} L^j = {} & \sum_{t=1}^{T} \sum_{s=1}^{S_j} \pi_s \left[\rho_t^{\text{D}} P^{\text{D}}_{j,t,s} + C_j^{\text{DG}} P^{\text{DG}}_{j,t,s} + C^{\text{IL}}_{j,t} P^{\text{IL}}_{j,t,s} \right] - \sum_{i=1}^{I} C^i_{j,t,s} \lambda^i_{j,t,s} \\ & - \lambda^{17}_{j,t,s} \left(E_{j,t,s} - E_{j,t-1,s} - \eta_j^{\text{ch}} P^{\text{ch}}_{j,t,s} + \frac{P^{\text{dch}}_{j,t,s}}{\eta_j^{\text{dch}}} \right) - \lambda^{18}_{j,t,s} \left(E_{j,t,s} - E^{\text{ini}}_j - \eta_j^{\text{ch}} P^{\text{ch}}_{j,t,s} + \frac{P^{\text{dch}}_{j,t,s}}{\eta_j^{\text{dch}}} \right) \\ & - \lambda^{19}_{j,t,s} \left(P^{\text{WT}}_{j,t,s} + P^{\text{DG}}_{j,t,s} + P^{\text{IL}}_{j,t,s} + P^{\text{D}}_{j,t,s} + P^{\text{dch}}_{j,t,s} - P^{\text{demand}}_{j,t,s} - P^{\text{ch}}_{j,t,s} \right) \end{aligned} \tag{61}$$